Impedance Magnitude Measurement from a Resistive Bridge

J. L. MASSEY, MEMBER, IEEE, AND L. V. AUTH, JR., MEMBER, IEEE

Abstract—**It is shown that the familiar resistive bridge can be used to measure the magnitude of an unknown impedance. In order for this conclusion to hold, the bridge must have equal ratio arms. The measured voltage is then a minimum when the variable resistance in the bridge is equal to the magnitude of the unknown resistance. The sensitivity, however, decreases as the reactive component of the unknown impedance increases. The analytical tools used are well within the reach of the sophomore student in electrical engineering.**

Introduction

The use of the four-element, or wheatstone bridge, circuit to measure an unknown impedance has become a classical measurement technique in electrical engineering.[1] The general form of such a bridge with resistances in all arms except that containing the unknown element is shown in Fig. 1, where the measuring device is assumed to be a high impedance device such as a vacuum-tube-voltmeter. The meter voltage V_m is easily seen to be given by

$$V_m = \left(\frac{R_2}{R_1 + R_2} - \frac{Z}{Z + R_s}\right) V_s \tag{1}$$

from which it can be seen that the bridge can be balanced, i.e., $V_m = 0$, if and only if Z is entirely resistive. We will show, however, that useful information can be obtained, even when this is not the case, from the value of R which causes the bridge to be as nearly balanced as possible, i.e., which minimizes $|V_m|$, the magnitude of the meter reading. The following analysis was stimulated by questions from students in the sophomore electrical networks laboratory as to what was the significance of their bridge measurements at frequencies where the unknown impedance displayed a substantial reactive component.

Conditions for Minimizing $|V_m|$

It will be convenient to let

$$= \frac{R_2}{R_1 + R_2} \tag{2}$$

and

$$\beta = \frac{V_m}{V_s} \tag{3}$$

so that (1) may be rewritten

$$\beta = \frac{Z(\rho - 1) + \rho R_s}{Z + R_s}. \tag{4}$$

Since V_s is a constant, the bridge is here assumed to be driven by an ac source of constant amplitude, minimization of $|\beta|$ is the same as minimization of $|V_m|$.

In the special case of an equal-ratio-arm bridge, $R_1 = R_2$, we have $\rho = \frac{1}{2}$ and (4) becomes

$$\beta = \frac{1}{2} \frac{R_s - Z}{R_s + Z}. \tag{5}$$

The locus of β, as R_s is varied, is a circle according to the well-known circular-loci theorem.[2] Two points on this circle are easily obtained.

Manuscript received August 16, 1966.
J. L. Massey is with the Department of Electrical Engineering, University of Notre Dame, Notre Dame, Ind.
L. V. Auth, deceased, was formerlv with the Department of Electrical Engineering, University of Notre Dame. Ina.

[1] B. Hague, *A-C Bridge Methods*, 5th ed. rev. London: Pitman & Sons, 1957.
[2] H. H. Skilling, *Electrical Engineering Circuits*, 2nd ed. New York: Wiley, 1965, p. 401.

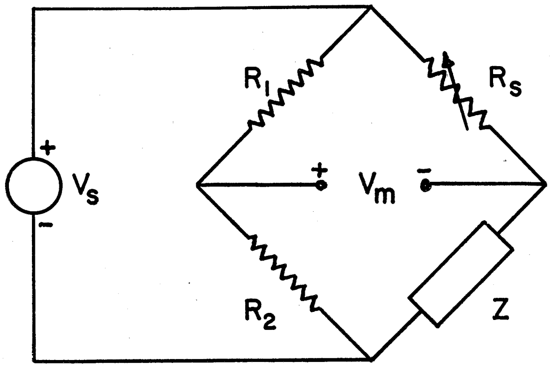

Fig. 1. Resistive wheatstone bridge circuit.

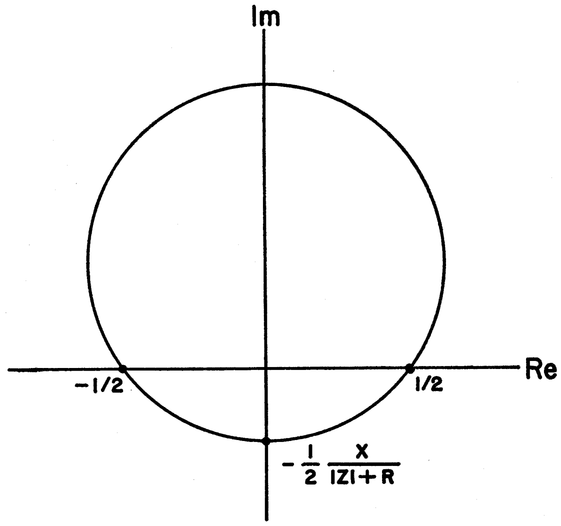

Fig. 2. Locus of $\beta = V_m/V_s$ as R_s is varied for $R_1 = R_2$.

When R_s equals 0 and $+\infty$, we find that β assumes the real values $-\frac{1}{2}$ and $+\frac{1}{2}$, respectively. One further point completely specifies the locus and is conveniently found by setting $R_s = |Z|$. Letting $Z = R + jX$, we find in this case that

$$\beta = -\frac{1}{2} j \frac{X}{|Z| + R}. \tag{6}$$

Hence, β is purely imaginary at this point and the entire locus is as shown in Fig. 2.

There are several points to be made from the locus in Fig. 2. Clearly, $|\beta|$ is a minimum at the point given by (6), unless $|\beta| = \frac{1}{2}$ at this point, in which case $|\beta|$ always has the constant value of $\frac{1}{2}$. Inspection of (6) and the fact that $|Z| = R^2 + X^2$ shows that this can happen when and only when $R = 0$, i.e., only when the unknown element is entirely reactive. Except for this case then, the unique minimum of $|\beta|$, and hence of $|V_m|$, occurs when $R_s = |Z|$ since this is the condition for (6). Moreover, it can be seen that the minimum of $|\beta|$ is less pronounced as X increases relative to R, i.e., the sensitivity of the bridge decreases.

Conclusions

It has been shown above that when $R_1 = R_2$, then the value of R_s which minimizes the amplitude of the meter voltage in the bridge circuit of Fig. 1 occurs when R_s is equal to the magnitude of the unknown impedance, except in the degenerate case when Z is entirely reactive. Although sensitivity decreases as Z becomes more reactive, the circuit is quite practical for elements with a reactive angle as large as 45°. The assumption that $R_1 = R_2$ which reduced (4) to the convenient form of (6) is necessary for this result to hold. A more lengthy analysis shows that for arbitrary ρ, the minimum value of $|\boldsymbol{V}_m|$ occurs when

$$\rho R R_s^2 + |\boldsymbol{Z}|^2(2\rho - 1)R_s + R|\boldsymbol{Z}|^2(\rho - 1) = 0 \qquad (7)$$

from which it is difficult to extract any useful information. Incidentally, the analysis here for $\rho = \frac{1}{2}$ justifies the intuitive measurement technique of setting $R = R_s$ for a bridge of the type in Fig. 1, when perfect balance cannot be obtained because of a small reactive component in $\boldsymbol{Z}$. The simple analysis presented here is easily within the reach of sophomore students. The use of these concepts in a bridge experiment leads to a more lively and interesting experiment than is provided by the usual dc wheatstone bridge.

Reprinted from IEEE TRANSACTIONS
ON *EDUCATION*
Volume E-10, Number 1, March, 1967
pp. 50-51