Therefore for an input state $\rho = \Sigma_{n=0}^{N}\lambda_n E_n$ $(0 \leqslant N \leqslant \infty)$ with $\Sigma_n \lambda_n = 1$ and $\lambda_k \neq \lambda_j$ $(k \neq j)$, the compound state and the trivial compound state introduced in Section IV are given by

$$\sigma_E = \Sigma_n \lambda_n E_n \otimes \Lambda^* E_n = \Sigma_{n=0}^{N}\Sigma_{j=0}^{n}\lambda_n |c_j^n|^2 E_n \otimes \theta_j,$$

$$\sigma_0 = \Sigma_n \lambda_n E_n \otimes \Sigma_k \lambda_k \Lambda^* E_k = \Sigma_{n,k}\Sigma_{j=0}^{k}\lambda_n \lambda_k |c_j^k|^2 E_n \otimes \theta_j,$$

where θ_j is defined by $\theta_j = |\Phi_j\rangle\langle\Phi_j| \in T(\mathcal{H}_2)_{+,1}$. Thus we can calculate the mutual information as follows:

$$\begin{aligned} I(\rho;\Lambda^*) &= -\Sigma_n \lambda_n \log \lambda_n - \Sigma_{j=0}^{N}\left(\Sigma_{n=j}^{N}\lambda_n |c_j^n|^2\right) \\ &\quad \cdot \log\left(\Sigma_{n=j}^{N}\lambda_n |c_j^n|^2\right) + \Sigma_n \Sigma_{j=0}^{n}\left(\lambda_n |c_j^n|^2\right)\log\left(\lambda_n |c_j^n|^2\right) \\ &= \Sigma_{j=0}^{N}\Sigma_{n=j}^{N}\lambda_n |c_j^n|^2 \log\left(|c_j^n|^2 / \left(\Sigma_{n=j}^{N}\lambda_n |c_j^n|^2\right)\right). \end{aligned}$$

This result is also obtained from the Shannon's formula with an initial probability $\{\lambda_n\}$ and the conditional probability $\{|c_j^n|^2\}$ given above. As the above model is very simple, this conditional probability can be calculated [20] without introducing a channel Λ^* explicitly. Once the conditional probability is known (e.g., when every eigenvalue of ρ is nondegenerate and Λ^* is given explicitly, it can be obtained by taking proper CONS's at the input and output systems), Shannon's expression for the mutual information is equivalent to ours. Without formulating a channel and a compound state precisely, neither have we a general rule to compute the mutual information (the joint probability) nor can we study quantum communication processes in general or the properties of a channel and the mutual information in particular.

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Capacity of Interconnected Ring Communication Systems with Unique Loop-Free Routing

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Abstract*—*Capacity* is defined for a given distribution of offered traffic as the maximum rate with which packets can be sent with finite delay through the network by appropriate routing. It is shown how capacity depends on the traffic characteristics and on the topology of ring communication systems interconnected so as to provide unique loop-free routing. First, the traffic conditions are given under which the capacity of a *single ring* attains its maximum and its minimum. For the case of *uniform traffic* it is shown that the capacity is equal to twice the minimum capacity. Then it is shown that, for uniform traffic, the capacity relative to a single ring communication system can be increased by as much as 33.3 or 80 percent when the stations are split up into two or three separate rings, respectively, interconnected to give unique loop-free routing. Exact formulas are given for the capacity of systems with an arbitrary number of stations split up into an arbitrary number of separate rings interconnected to give unique loop-free routing. Finally, it is shown that connecting *local rings* through a star network with a central switching node is particularly useful when stations can be segregated into local groups of stations which often communicate with stations from the same local group but only rarely with stations from the other groups. Exact formulas are given to calculate the capacity of such interconnected ring communication systems.

I. Introduction

In a communication network with many senders and/or many receivers, the ultimate limit of reliable communications can be specified by a *capacity region* with one coordinate for each source-destination pair. Alternatively, one can consider "capacity" to be a scalar which depends on the *offered traffic distribution*, i.e., on what fraction of the total traffic originates at station i and is destined for station j for all $i \neq j$. For a given offered traffic distribution, capacity is the upper limit of the total offered traffic rate such that there exist protocols that deliver this traffic with finite average delay. If the total offered traffic exceeds

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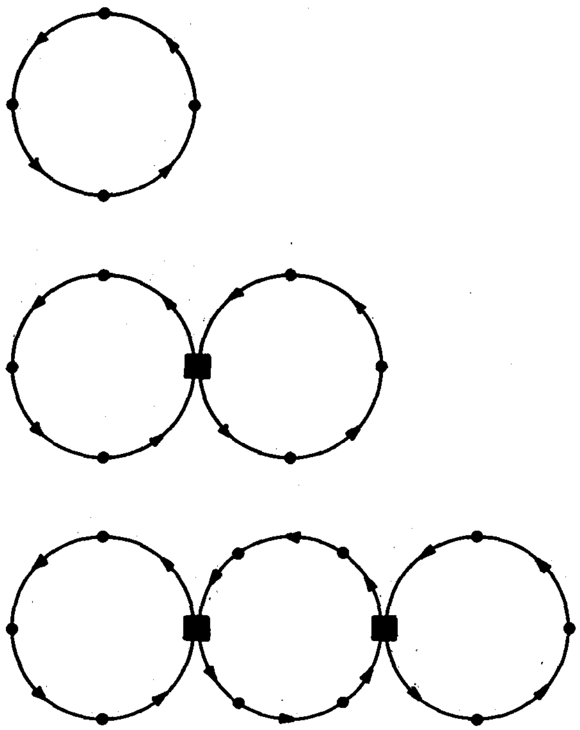

Fig. 1. Examples of communication networks with unique loop-free routing.

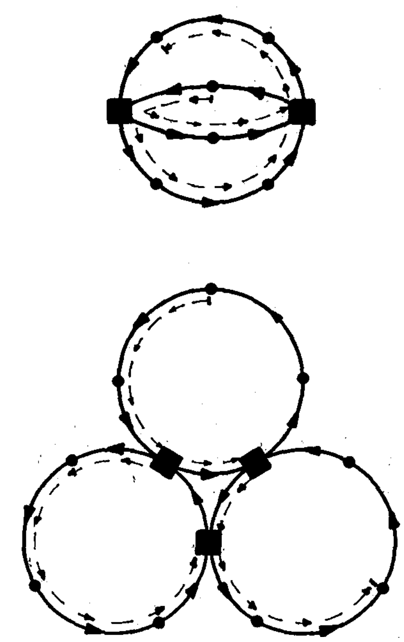

Fig. 2. Examples of communication networks with nonunique routing.

capacity, then the average packet delay must be infinite because the network buffers hold a store of packets that almost surely increases without bound as further traffic arrives; here we assume that all buffers are of infinite length so that no packets are lost because of buffer overflow.

The capacity of general networks has been discussed, for example, in [1]–[6], but the interesting and easily analyzed class of networks with *unique loop-free routing* has apparently not been singled out previously. In Section II, we find the capacity region of this class of networks and show how capacity depends on the *offered traffic vector*, a quantity that describes the offered traffic characteristics. In Section III, we discuss the capacity of the simplest network with unique loop-free routing, namely, a single ring with unidirectional links. In Sections IV and V, we discuss the capacity of connected ring systems with exchange nodes for the uniform and certain nonuniform traffic distributions.

II. The Capacity Region

We consider communication systems with *m stations* (shown as nodes in Fig. 1) interconnected by identical unidirectional links to give *unique loop-free routing*; we assume that each link is an error-free delayless binary channel and hence has a capacity of 1 (bit per bit-time). The networks shown in Fig. 1 are typical examples of such communication systems. The *exchange nodes* (shown as squares) route the traffic from all incoming links to the appropriate outgoing links; they neither generate nor absorb any traffic. To illustrate what we mean when we say that the networks in Fig. 1 have unique loop-free routing we show, in Fig. 2, two examples of networks with nonunique routing. In these networks there are, as illustrated by the dashed lines, cases where an exchange node can route the incoming traffic to more than one possible outgoing link without creating a closed-loop within the resulting path.

We assume that each station comprises a sender S and a receiver R. Under the assumption here and hereafter that the senders do not transmit to receivers at the same station, there are $n = m(m-1)$ sender–receiver pairs (S_i, R_j) with $i \neq j$ to be considered.

We define the n-dimensional *offered-traffic vector*

$$f = R\boldsymbol{\theta}, \tag{2.1}$$

where R is the *total offered traffic* (in bits per bit-time) and where

$$\boldsymbol{\theta} = [\theta_{1,2}, \cdots, \theta_{1,m}, \theta_{2,3}, \cdots, \theta_{2,1}, \cdots, \theta_{m,1}, \cdots, \theta_{m,m-1}] \tag{2.2}$$

is the *traffic distribution vector* for which $\theta_{i,j} \geqslant 0$ and

$$\sum_{\substack{i,j \\ i \neq j}} \theta_{i,j} = 1. \tag{2.3}$$

The component $\theta_{i,j}$ is the fraction of the total offered traffic that originates at station i and is destined for station j. The quantity

$$f_{i,j} = R\theta_{i,j} \tag{2.4}$$

is the time-average offered traffic (in bits per bit-time) from sender S_i to receiver R_j. Obviously,

$$f_{i,j} \geqslant 0, \qquad \text{all } i, j. \tag{2.5}$$

Equations (2.3) and (2.4) also imply the obvious relation

$$R = \sum_{i,j} f_{i,j}. \tag{2.6}$$

Any vector f that satisfies (2.5) is a possible offered traffic vector.

Because of the unique loop-free routing, the traffic that must flow through link k, if all packets are to reach their destination, is given by

$$F_k = \sum_{(i,j) \in S_k} f_{i,j}, \tag{2.7}$$

where S_k is the set of all origin-destination pairs (i, j) such that the unique loop-free path from station i to station j traverses link k. What makes unique routing networks amenable to simple analysis is that the offered traffic uniquely determines the link flows according to (2.7) for all protocols that produce loop-free routes and that deliver all packets to their destinations; protocols not having these two properties are of no interest.

Any vector f that satisfies (2.5) and $F_k \leqslant 1$, $1 \leqslant k \leqslant m$, is said to be a *feasible offered traffic vector*. (In general networks, an offered traffic vector is said to be feasible (see, e.g., [1]) if there exists a routing for the flows such that the total flow in each link for this routing does not exceed the capacity of that link; our usage of "feasible" is consistent with this general usage.)

We now define the region $\mathbb{C}$ to be the set of all feasible offered traffic vectors, i.e.,

$$\mathbb{C} = \left[f: f_{i,j} \geq 0 \text{ all } i \neq j \text{ and } F_k \leq 1 \text{ all } k \right]. \tag{2.8}$$

The following observation establishes that $\mathbb{C}$ is in fact the capacity region of the network considered as a multi-user channel.

If the flow vector f lies within the outer boundary of the region $\mathbb{C}$ (i.e., if $F_k < 1$ for all links k), then a simple store-and-forward first-in first-out protocol at each node will serve all packets with finite average delay when the packet arrival processes are independent and memoryless and the second moment of the packet length is finite. Conversely, if the flow vector f lies beyond the outer boundary of the capacity region (i.e., if $F_k > 1$ for at least one link k) then no protocol can serve all packets with finite average delay since the average delay of packets traversing this link must be infinite regardless of the nature of the packet arrival process or the packet length distribution. We remark that those f that lie on the outer boundary of $\mathbb{C}$ (i.e., those f with $F_k = 1$ for some links k) can be served with finite average packet delay only in special cases such as periodic arrivals of packets of constant length. When packet arrivals are memoryless and independent, however, the average delay for packets traversing a link with $F_k = 1$ will be infinite regardless of the serving protocol.

We now introduce the notation

$$C(\theta) = \max\left[R: f \in \mathbb{C} \text{ and } f = R\boldsymbol{\theta} \right] \tag{2.9}$$

to denote the capacity as a function of the traffic distribution vector $\boldsymbol{\theta}$. In the case of networks with unique loop-free routing, $C(\boldsymbol{\theta})$ is equal to that value of R which solves

$$F_{k^*} = 1, \tag{2.10}$$

where k^* is a value of k that maximizes F_k. We shall call (2.10) the *most-congested-link equation*.

III. Single Ring Networks

Here we consider the case of single ring networks. We first assume that $\theta_{i,i+j}$ has the same value for all i (where here and hereafter, subscripts formally greater than m are understood to be reduced by m). This implies that the flows between S_i and R_{i+j}, $1 \leq i \leq m$, $1 \leq j \leq m-1$, depend only on the distance j between sender and receiver, and, also, that all senders S_i, $1 \leq i \leq m$, generate the same total amount of traffic. We shall call such a traffic distribution *symmetric*, and we write

$$\tilde{\theta}_j = m\theta_{i,i+j} \tag{3.1}$$

to denote the fraction of the total traffic destined j links from its origin. The traffic that must flow through link k in a single ring is given in general by

$$F_k = R \sum_{j=1}^{m-1} \sum_{i=0}^{(m-1)-j} \theta_{k-i,k+j}. \tag{3.2}$$

Because of (3.1), (3.2) reduces for symmetric traffic to

$$F_k = \frac{R}{m} \sum_{i=1}^{m-1} i\tilde{\theta}_i \tag{3.3}$$

which, not surprisingly, is independent of k. Thus, the most-congested link-equation (2.10) gives the *symmetric capacity* as

$$C(\boldsymbol{\theta}) = \frac{m}{\sum_{i=1}^{m-1} i\tilde{\theta}_i}. \tag{3.4}$$

From (3.5), we see that symmetric capacity $C(\boldsymbol{\theta})$ attains its maximum value $C_{\max}$ when the offered traffic is such that each

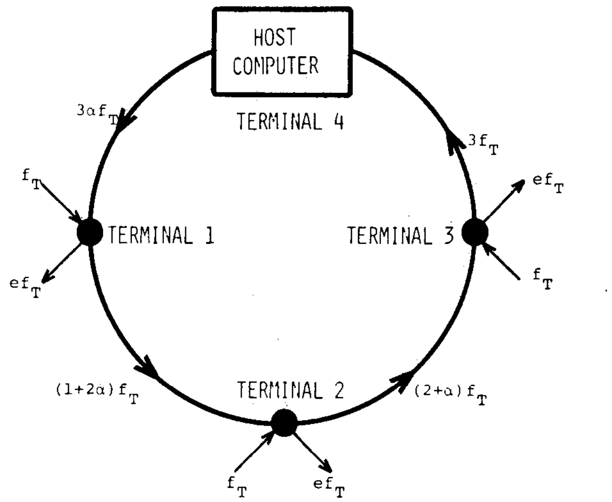

Fig. 3. Ring communication systems with terminal-to-host and host-to-terminal traffic.

sender transmits only to its nearest neighboring receiver, i.e., when $\tilde{\theta}_1 = 1$ and $\tilde{\theta}_i = 0$ for $i > 1$. In this case, each link carries the flow from only one sender and (3.4) yields

$$C_{\max} = m. \tag{3.5}$$

From (3.4), we see further that the symmetric capacity achieves its minimum $C_{\min}$ when each sender transmits only to its most distant receiver, i.e., when $\tilde{\theta}_{m-1} = 1$ and $\tilde{\theta}_i = 0$ for $i < m-1$, namely,

$$C_{\min} = \min_{\tilde{\boldsymbol{\theta}}}(\tilde{\boldsymbol{\theta}}) = \frac{m}{m-1}. \tag{3.6}$$

In this case, each link carries the flow from $m-1$ senders. For $m \gg 1$, $C_{\min}$ is approximately 1. In other words, when all senders transmit only to their most distant destinations, ring capacity is approximately equal to the capacity of a single link between two stations. The maximum rate with which each station can send is approximately $1/m$.

In real applications, the flows will in general be such that the symmetric ring capacity is between $C_{\max}$ and $C_{\min}$ as given by (3.5) and (3.6), respectively. Unfortunately, however, it will usually be much closer to $C_{\min}$ than to $C_{\max}$. Consider, for example, the case of *uniform traffic* which we define as the situation when the traffic is symmetric and

$$\tilde{\theta}_1 = \tilde{\theta}_2 = \cdots = \tilde{\theta}_{m-1} = \frac{1}{m-1}, \tag{3.7}$$

i.e., when each sender sends with the same rate to each of the $m-1$ receivers at the other stations. Equation (3.4) now gives the capacity, which we denote by C_{unif}, as

$$C_{\text{unif}} = 2, \tag{3.8}$$

irrespective of the number of stations, i.e., the ring as a whole can carry only twice as much traffic as a single link or, equivalently, that each link carries exactly half of the total traffic.

(A minor modification of the above arguments shows that the maximum achievable rate *for protocols that allow packets to be removed from the ring only when they reach their original sender* is at most 1 in a single ring network regardless of the traffic distribution vector $\boldsymbol{\theta}$. Thus, with uniform traffic, such end-to-end protocols can at best operate at 50 percent of capacity.)

To illustrate that the symmetric and uniform analysis can yield substantially incorrect performance estimates for other traffic distributions, consider a ring used for *terminal-to-host* and *host-to-terminal* computer traffic as depicted in Fig. 3. Terminals $1, 2, \cdots, m-1$ each transmit with rate f_T only to station m, the

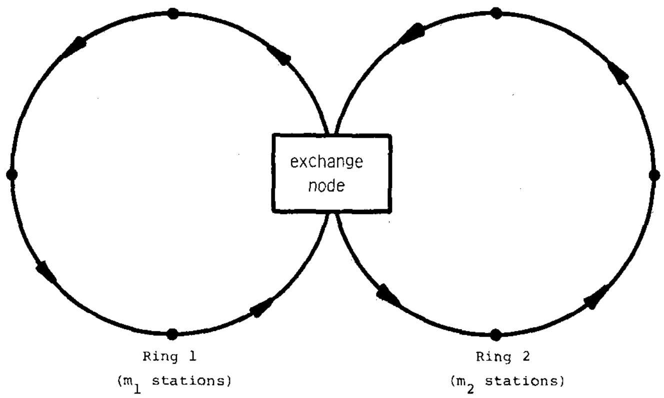

Fig. 4. Communication systems consisting of two rings which are interconnected by an exchange node.

host computer. The host computer sends return messages at the same rate f_H to each of the terminals, and there is no other traffic. Let α denote the *ratio of host-to-terminal traffic*, i.e.,

$$f_H = \alpha f_T. \tag{3.9}$$

For $\alpha \geqslant 1$ (as in an *inquiry system* where a short message from a terminal usually causes the host computer to send a large amount of data such as a full screen of text back to the terminal) the most congested link is that from the host computer to the first downstream station; its flow is

$$F_1 = \frac{R}{m(1+\alpha)} ma = R\frac{\alpha}{1+\alpha}. \tag{3.10}$$

The capacity, which we now denote as C_α, is found from (3.4) to be

$$C_\alpha = 1 + \frac{1}{\alpha}, \qquad \alpha \geqslant 1, \tag{3.11}$$

which is equal to 2 (as in the case of uniform traffic) when $\alpha = 1$ but decreases rapidly to $C_{\min} = 1$ as α increases.

For $\alpha < 1$ (as in a *data collection system* where all terminals send their data to the host which sends only short acknowledgment messages back to the terminals) the flow is maximum in the link just before the host computer; its flow is

$$F_m = \frac{R}{m(1+\alpha)} m = \frac{R}{1+\alpha}. \tag{3.12}$$

The capacity is found from (3.4) to be

$$C_\alpha = 1 + \alpha, \qquad 0 \leqslant \alpha < 1. \tag{3.13}$$

Again, ring capacity is less than 2; for $\alpha \ll 1$, C is reduced to almost $C_{\min} = 1$. Thus, either for $\alpha \ll 1$ or $\alpha \gg 1$, the capacity of the ring is only about half its uniform capacity.

IV. Capacity of Interconnected Ring Systems with Uniform Traffic

When, in a given application, the ring capacity would not be sufficient for the desired offered load, one must either increase the capacity of the individual links—or choose a network with an alternative topology which reduces the flow through some links. In this section we consider a special case of the latter approach which is particularly useful to increase the capacity of an existing ring communication system when either more stations are to be connected or when, in the course of time, stations have more data to send than when the system was initially planned and built.

We first consider a system of two rings which are connected by an exchange node (Fig. 4). Of the total of m stations, m_1 are connected to ring 1 and $m_2 = m - m_1$ are connected to ring 2. We assume *uniform traffic*, i.e., that each station i sends with the same rate

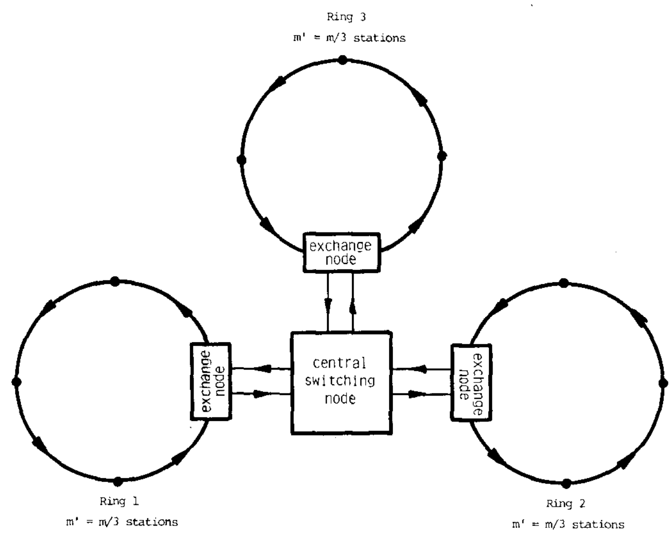

Fig. 5. Communication system consisting of three rings which are interconnected by a star network.

$$f_{i,j} = \frac{R}{m(m-1)} \tag{4.1}$$

to all other stations $j \neq i$. To find capacity in this case, we note from inspection of Fig. 4 that the traffic which must flow through any link in ring 1 is the same as the traffic $R/2$, which would flow through that link if all stations had been connected to form one ring less half of the traffic $R_{2,2}$ that both originates in and is destined for stations in ring 2. Thus,

$$F_k^{(1)} = \frac{R}{2} - \frac{R_{2,2}}{2}, \tag{4.2}$$

where

$$R_{2,2} = \frac{m_2(m_2-1)}{m(m-1)} R. \tag{4.3}$$

Combining these equations gives

$$F_k^{(1)} = \frac{Rm_1}{m(m-1)}\left[\frac{m_1-1}{2} + m_2\right]. \tag{4.4}$$

Assuming $m_1 \geqslant m_2$, the links in ring 1 become completely used before the links in ring 2, so (2.10) yields

$$C_{\text{unif}}^{(2)} = \frac{2m(m-1)}{m^*(2m - m^* - 1)} \tag{4.5}$$

where we also include the case $m_2 < m_1$ by defining

$$m^* = \max(m_1, m_2). \tag{4.6}$$

The capacity gain over a single ring (which has $C_{\text{unif}} = 2$) is largest when $m_1 = m_2 = m/2$; in this case (4.5) reduces to

$$C_{\text{unif}}^{(2)} = \frac{8(m-1)}{3m-2}, \tag{4.7}$$

which quickly approaches 8/3 as the number of stations m increases. In other words, the best that can be achieved (in the case of uniform traffic) is a 33.3 percent increase in total capacity when the number of stations in a given ring system is split up into two rings which are connected by an exchange node. If this increase is not sufficient, the stations must be split up further, e.g., into three separate rings. Such a splitting is shown in Fig. 5 where the exchange nodes are interconnected by a star network. We assume that the links between the exchange nodes and the

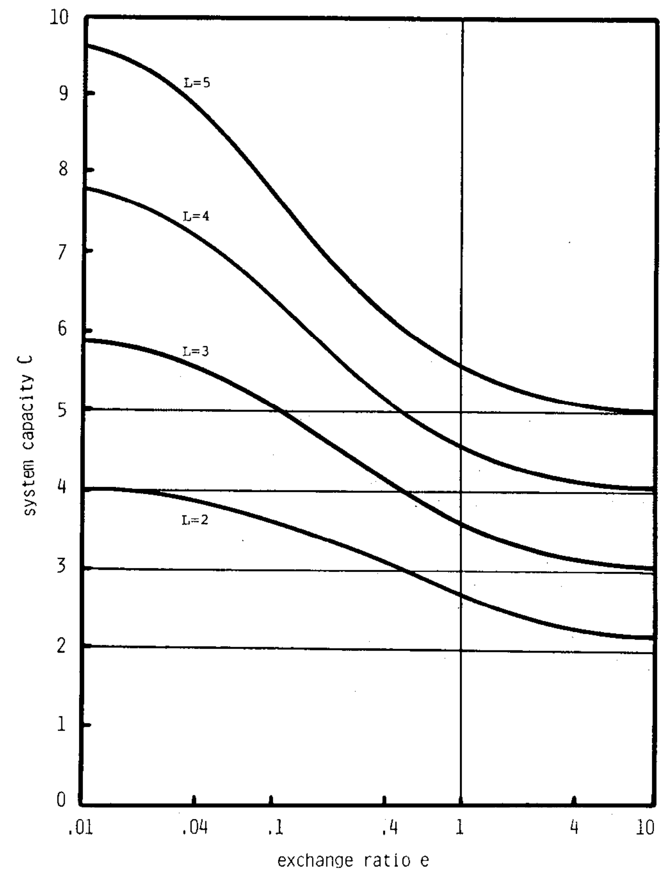

Fig. 6. Capacity of a system with $m \to \infty$ stations which are equally split into L separate rings which are interconnected by a star network (5.3).

central switching node in the star network have ample capacity to guarantee that they do not become bottlenecks.

In general, when m stations with uniform traffic (where m is a multiple of L) are equally split into L rings connected through a star network, the resulting uniform capacity can be shown to be

$$C_{\text{unif}}^{(L)} = \frac{2L^2(m-1)}{m(2L-1)-L}. \tag{4.8}$$

As m increases for fixed L, $C_{\text{unif}}^{(L)}$ approaches $2L^2/(2L-1)$ which is very nearly L for even modestly large values of L.

V. Capacity of Connected Ring Systems with Nonuniform Traffic

In the previous section we saw that ring communication systems with uniform traffic have a higher capacity when the stations are split up into a number of local rings which are connected through a star network. In this section we show that, in applications where the stations can be grouped into stations which often communicate with stations from the same group but only rarely with stations from other groups, the capacity advantage of such connected ring systems compared to a single ring is even greater.

Suppose that m stations are split equally into L ($L \geqslant 2$) rings where each ring has $m' = m/L$ stations which send with the same rate to all other stations in the same ring and with a fraction e of this rate to all stations in all the other rings. We call e the *exchange ratio* of the offered traffic. All links then carry the same traffic and it is straightforward to show that (2.10) gives the capacity as

$$C_{\text{nonunif}}^{(L)} = \frac{2L[m(1+e(L-1))-L]}{m[1+2e(L-1)]-L}. \tag{5.1}$$

For $e = 0$, i.e., when there is no exchange traffic, we have L separate local rings with uniform traffic. Because each of these local rings has a capacity of 2, the system capacity must be $2L$. At the other extreme, when e is large we have, in the limit,

$$\lim_{e\to\infty} C_{\text{nonunif}}^{(L)} = L. \tag{5.2}$$

This is illustrated in Fig. 6 where

$$\lim_{m\to\infty} C_{\text{nonunif}}^{(L)} = 2L\frac{1+e(L-1)}{1+2e(L-1)} \tag{5.3}$$

is plotted versus e for $L = 2, 3, 4$, and 5 separate local rings.

VI. Conclusion

We have shown how capacity, the maximum rate with which packets can be sent through a ring communication system, depends on how the senders and receivers are physically located relative to each other. For the case of uniform traffic, i.e., when each sender sends with the same rate to all receivers at the other stations, the capacity of a single ring is equal to twice the capacity of a transmission link between two stations.

A way to increase the capacity of a single ring without increasing the capacity of the individual transmission links is to split the system up into separate rings. For uniform traffic, the capacity of a ring communication system with two separate identical rings connected through an exchange node is approximately 2.6 times the capacity of a single transmission link. With three rings, the capacity is approximately 3.6 times the capacity of a single transmission link. Exact formulas have been given for the general case with an arbitrary number of stations and arbitrary numbers of individual rings.

Ring communication systems with separate local rings which are connected through a star network with a central switching node are particularly well suited to applications where stations can be segregated into local groups of stations which often communicate with stations from the same local group but only rarely with stations from the other groups. When there is much exchange traffic between the individual rings, system capacity is approximately equal to half the sum of the capacities of the individual rings. When, however, there is little exchange traffic, system capacity approaches the sum of the capacities of the individual rings.

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Correction to "A Simple General Binary Source Code"

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We wish to correct a mistake in the description of the decoding algorithm for the binary source code of the above correspon-

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