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The following correction will appear in *Problemy Peredachi Informatsii*:

Correction to "Causal Interpretations of Random Variables"[1]

We are very grateful to Milan Studeny of the Institute of Information Theory and Automation, Academy of Sciences of the Czech Republic, for pointing out that Proposition 1 is not true for our definition of "equivalence" of causal interpretations. None of the other results in the paper depended on this definition and all are valid as stated. For Proposition 1 to hold, our definition should be modified to: the causal interpretation $(X_{i_1}, X_{i_2}, \ldots X_{i_N})$ is *equivalent* to the causal interpretation $(X_{j_1}, X_{j_2}, \ldots X_{j_N})$ if, whenever some X_k is causally directly prior to some X_n with respect to one of these causal interpretations, then X_k appears before X_n in the list of random variables in the other causal interpretation.

James L. Massey

Further Explanation of the Correction

That our earlier definition of "equivalence" of causal interpretations was inadequate is shown by the following simple example:

Let X_1 be a random variable taking on each of the values 0, 1, 2 and 3 with probability $\frac{1}{4}$, and let $X_2 = 2 \odot X_1$ and $X_3 = X_1 \oplus 2$ where the product and sum are modulo 4. Suppose that the convention for obtaining unique reduced-conditioning expressions for entropy is first to remove X_1 if possible, then to remove X_2 if possible, and finally to remove X_3 if possible. Then, for the causal interpretation (X_1, X_2, X_3), the reduced-conditioning expressions for the terms in the causal-order expansion of $H(X_1X_2X_3)$ are $H(X_1)$, $H(X_2|X_1)$ and $H(X_3|X_1X_2) = H(X_3|X_1)$. Hence, the only random variable causally directly prior to X_2 is X_1 and the only one causally directly prior to X_3 is also X_1. By our definition of "equivalence", the causal interpretation (X_1, X_3, X_2) is thus equivalent to the causal interpretation (X_1, X_2, X_3) because X_1 appears before both X_2 and X_3 in the list (X_1, X_3, X_2). However, for the causal interpretation (X_1, X_3, X_2), the reduced-conditioning expressions for the terms in the causal-order expansion of $H(X_1X_2X_3)$ are $H(X_1)$, $H(X_3|X_1)$, and $H(X_2|X_1X_3) = H(X_2|X_3)$, where the last equality follows from the fact that $X_2 = 2 \odot X_3$ so that X_1 can be removed from the conditioning. Hence, for the causal interpretation (X_1, X_3, X_2), the only random variable causally directly prior to X_3 is X_1 as before, but the only one causally directly prior to X_2 is now X_3 rather than X_1. It follows that the causality graphs of these two causal interpretations are different, in contradiction to our Proposition 1.

With modified definition of equivalence given in the above correction, Propo-

[1] *Problemy Peredachi Informatsii*, vol. 32, no. 1, pp. 131-136, Jan.-March, 1996.

sition 1 as stated in our paper is easily proved. For suppose that $(X_{i_1}, X_{i_2}, \ldots X_{i_N})$ and $(X_{j_1}, X_{j_2}, \ldots X_{j_N})$ are equivalent causal interpretations, that $X_{i_k} = X_{j_n} = X$, and that the reduced-conditioning expressions for $H(X|X_{i_1} \ldots X_{i_{k-1}})$ and $H(X|X_{j_1} \ldots X_{j_{n-1}})$ are $H(X|X_C)$ and $H(X|X_D)$, respectively. The new definition of equivalence then requires that both X_C and X_D be subvectors of both $(X_{i_1}, \ldots, X_{i_{k-1}})$ and $(X_{j_1}, \ldots, X_{j_{n-1}})$, and hence it must be true, by the uniqueness of reduced-conditioning expressions for entropy, that $X_C = X_D$. Thus, the random variables causally directly prior to X with respect to the causal interpretation $(X_{i_1}, X_{i_2}, \ldots X_{i_N})$ are the same as the random variables causally directly prior to X with respect to the causal interpretation $(X_{j_1}, X_{j_2}, \ldots X_{j_N})$ so that the causality graphs of these two causal interpretations indeed coincide.