\[
\mathcal{L}(0,1) + \mathcal{L}(0,1) + \mathcal{L}(1,0) = (1,1,1) + (1,1,1) + (0,0,1) = (1,1,0)
\]

The capacity region for the two-way relay with two helper nodes is given by

\[
\{(x, y, z) \mid x \geq 0, y \geq 0, z \geq 0, x + y + z = \text{capacity region}\}
\]

If the capacity region is not convex, we need to add a constraint for the total capacity.

\[
\begin{align*}
(1) & \quad (x, y, z) \quad \text{s.t.} \\
& \quad x + y + z \leq \text{total capacity}
\end{align*}
\]

I. Introduction

The problem of finding the capacity region for the two-way relay channel with two helper nodes is an important problem in wireless communication systems.
It remains only to show that $\lambda$ is indeed the capacity of the channel.

To compute the proof we need to introduce the concept of the WTC.

For $P \in \mathcal{P}(\mathcal{Z})$, let the function $f(P)$ be defined by

$$f(P) = \sum_{z \in \mathcal{Z}} P(z) \log \frac{1}{P(z)}.$$

Theorem 1: The capacity of the channel is given by

$$C = \max_{P \in \mathcal{P}(\mathcal{Y})} f(P).$$

Proof: Let $P, Q \in \mathcal{P}(\mathcal{Z})$ and $Q$ be any admissible distribution. Then

$$f(P) - f(Q) = \sum_{z \in \mathcal{Z}} P(z) \log \frac{P(z)}{Q(z)} = \sum_{z \in \mathcal{Z}} P(z) \log \left( \frac{1}{Q(z)} \right).$$

By convexity, this is maximized when $Q$ equals the uniform distribution.

In particular, if $P_{\text{opt}}$ is the optimal distribution, then

$$f(P_{\text{opt}}) = \max_{P \in \mathcal{P}(\mathcal{Y})} f(P).$$

This completes the proof of the theorem.

Remarks: The capacity of a channel is the maximum rate at which information can be transmitted over the channel without error. It is a fundamental concept in information theory.

The capacity formula can be expressed in terms of the mutual information between the input and output of the channel:

$$C = \max_{P \in \mathcal{P}(\mathcal{Y})} I(X;Y).$$

where $I(X;Y)$ is the mutual information between the input $X$ and output $Y$ of the channel.

Theorem 2 (Continuity of the Capacity): The capacity $C$ is a continuous function of the channel matrix $P$.

Proof: Suppose $P$ changes by a small amount $\epsilon$ to $P + \epsilon P'$. Then

$$I(X;Y) = \int p(x) \log \left( \frac{\sum_{y} q(y|x)p(x)}{q(y|x)} \right) dx,$$

where $q(Y|X)$ is the output probability distribution. Since the integral of a continuous function is continuous, the capacity $C$ is also continuous.

Example: Consider a binary symmetric channel (BSC) with crossover probability $p$. The capacity of the BSC is

$$C = 1 - H(p),$$

where $H(p) = -p \log p - (1-p) \log (1-p)$ is the binary entropy function.
In summary, the support of the theorem holds true for sufficiently large $n$. Furthermore, the theorem extends to scenarios where

\[ \left( \begin{array}{c} x_n \\ y_n \\ z_n \\ w_n \\ \end{array} \right) = \left( \begin{array}{c} x_n \\ y_n \\ z_n \\ w_n \\ \end{array} \right) \]

for all $n$. Hence, the theorem is valid for the entire sequence.

**References**

1. [Author et al., 2019]
2. [Another author, 2020]
3. [Yet another author, 2021]
Figure 2: Coding situation for the MTC.

Figure 3: MTC after the re-try channel ( After).