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Coding for Multiple Access Communications

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Abstract: An idealized code-division multiple-access (CDMA) system is used as a vehicle to illustrate a new approach to coding for multiple-access channels that aims to achieve reasonable efficiency while avoiding the prohibitive complexity of optimum coding schemes. The new approach combines "partial demodulation" to separate users into roughly independent groups with the creation of "virtual channels" that allow the decoder for each user to act as if that user were the only user of the system. The virtual channels are created by means of simple uniquely decodable codes for the noisy binary adder channel. The approach is motivated and explained by means of examples.

1 Introduction

Coding techniques for single-sender single-receiver channels have reached a state of maturity. By contrast, coding techniques for many-sender single-receiver channels, i. e., for multiple-access channels, are still in their infancy. The rapid growth of radio communications has created a strong practical need for good coding techniques for multiple-access channels in which the received signal is the sum of the transmitted signals. An especially interesting channel of this type is that characterizing code-division multiple-access (CDMA) systems. Coding for the CDMA channel is about as difficult as for any multiple-access channel. Most approaches to efficient coding for the CDMA channel die on the altar of practicality; the schemes are simply too complex to implement. This complexity stems in large part from the necessity to do joint decoding of all users–in general, the complexity of such a joint decoder grows exponentially with the number of users. The purpose of this paper is to suggest an approach to coding for multiple-access channels that offers the hope of yielding practical systems whose efficiency is not too much less than an optimum impractical system.

In the next section, we introduce the simple model of an idealized CDMA system that we will use. We then describe a technique that we call "partial demodulation" for this channel. The partial demodulator separates the received signal into one signal for each of many small groups of users, who subsequently ignore the signals for all other groups. This is the first of two steps toward a practical but reasonably efficient coding system. In Section 3, we show that coding for one user group reduces to coding for the noisy binary adder channel of multi-user information theory, which motivates our investigation in Section 4 of coding for that channel. Here we argue that the standard approach to coding for the noisy binary adder channel, viz., the design of powerful uniquely decodable codes, is misguided. Instead, as the second and final step of our approach, we propose using very simple codes of the uniquely decodable type in a manner that essentially decouples the users in each group, creating a "virtual channel" for each user on which conventional single-sender coding techniques can be used. Examples are used freely to explain the approach. Our intention here is to sketch the broad outlines of the new approach, not to paint a fully detailed canvas.

2 Code-Division-Multiple-Access Systems

For our purposes, the signal-space model of an idealized "L-chip synchronous CDMA channel" as shown in Fig. 1 will an adequate channel model. The model describes one use of a memoryless channel. Each of the K users, say user i, sends a symbol B_i to his corresponding modulator. We assume that B_i is a real-valued random variable with second moment $E[B_i^2] = E_c$, where E_c is the average chip energy. We further assume that the random variables B_1, B_2, ..., B_M are statistically *independent*, which is the essential difference between a multiple-access channel and a single-user channel. Using the usual signal-space representation with respect to any convenient basis of orthonormal waveforms, we can view the modulator as converting B_i into the signal vector $B_i\mathbf{s}_i$ where $\mathbf{s}_i = (s_{i1}, s_{i2}, ..., s_{iL})$ is an L-component *spreading sequence* with energy L and real components. Our requiring that the symbols B_i and the components of $\mathbf{s}_i$ be real numbers is equivalent to considering a baseband channel, but all our conclusions carry easily over to the passband case when these components are complex numbers. The energy of $\mathbf{s}_i$ is

$$\mathbf{s}_i^T\mathbf{s}_i = \sum_{n=1}^{L} s_{in}^2 = L \qquad (1)$$

(where the superscript ^{T} denotes transposition) and our normalization of this energy to L is

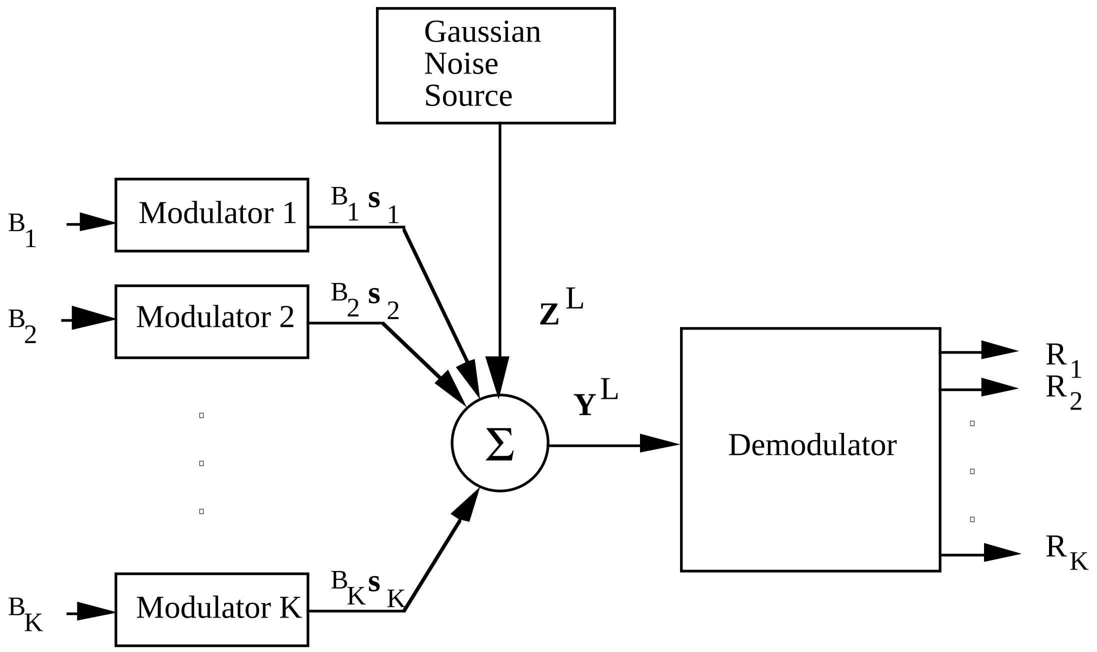

Fig. 1: Model of an L-chip synchronous CDMA channel.
($E[B_i^2] \le E_c$, $\mathbf{s}_i^T \mathbf{s}_i = L$, and $\mathbf{Z}^L$ is white Gaussian noise of variance $N_o/2$.)

motivated by the usual convention for CDMA spreading sequences that $s_{in} \in \{-1, +1\}$ for all n, by which we will also abide. The demodulator input is the received vector

$$\mathbf{Y}^L = \sum_{i=1}^{K} B_i \mathbf{s}_i + \mathbf{Z}^L \qquad (2)$$

where $\mathbf{Z}^L = (Z_1, Z_2, ..., Z_N)$ is *white Gaussian noise* (WGN), i.e., its components are independent zero-mean Gaussian random variables all with the same variance, $N_o/2$. We will be less explicit at present about the nature of the demodulator outputs, since these are at the choice of the designer of the modulation system.

It is well-known that an optimum demodulator (by any reasonable criterion) would be a bank of K matched filters whose outputs are the correlations of the received vector with the spreading sequences of the users, i. e., it would produce the outputs

$$R_i = \frac{1}{\sqrt{L}} \sum_{j=1}^{L} s_{ij} Y_j = \frac{1}{\sqrt{L}} \mathbf{s}_i^T \mathbf{Y}^L \qquad (3)$$

for i = 1, 2, ... K, where $\mathbf{Y}^L = (Y_1, Y_2, ..., Y_L)$, cf. [2], and where the factor multiplying the sum is included as a convenient normalization. The optimum decoder to be used with such a demodulator would be a *joint decoder* that has full responsibility for detecting the sequences of information bits from the K information sources. The problem with such a joint decoder is that in general its complexity grows exponentially with the number K of users so that it is impractical for interesting CDMA systems. In [1], we proposed instead to use a *partial demodulator* as shown in Fig. 2 that performs part of the user separation by partititioning the users into many "independent groups" and thus greatly simplifies the task of the decoder for each of these user groups.

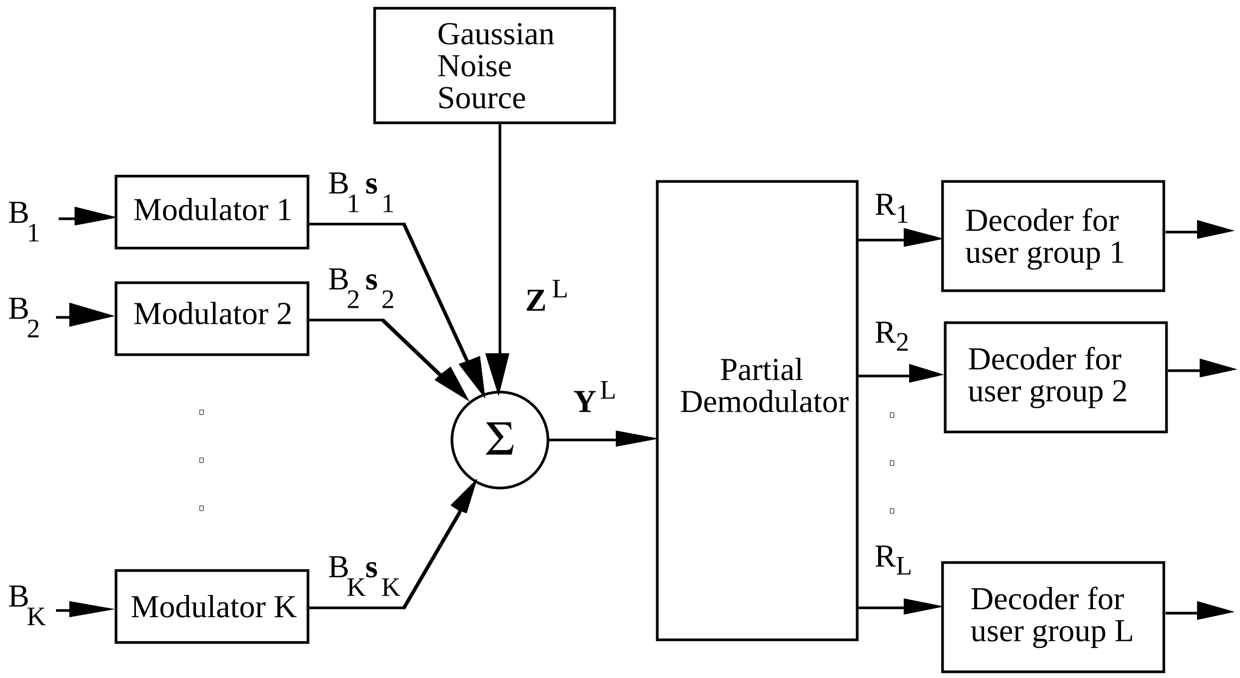

Fig. 2: Partial demodulation into signals for L groups of m users (K = mL)

For simplicity of notation, we assume that K = mL for some positive integer m and that the users are partitioned into L groups of m users each. We further assume that $\mathbf{s}_1, \mathbf{s}_2, ..., \mathbf{s}_L$ are now *mutually orthogonal sequences* (i.e., $\mathbf{s}_i^T\mathbf{s}_j = 0$ for $i \neq j$), each of energy L, and that *all users in the* i^{th} *user group* G_i u*se the same spreading sequence* $\mathbf{s}_i$. Now, suppose that the "partial demodulator" of Fig. 4 correlates each of the L sequences assigned to the user groups with the received sequence $\mathbf{Y}^L$ (which requires only $L = K/m$ correlations compared to K correlations for the full demodulator considered in Section 2). This partial demodulator then delivers to the decoder for the i^{th} user group the output

$$R_i = \frac{1}{\sqrt{L}} \mathbf{s}_i^T \mathbf{Y}^L = \sqrt{L} \sum_{j \in G_j} B_j + N_i \tag{4}$$

where we have used (2) and the mutual orthogonality of $\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_L$ and where

$$N_i = \frac{1}{\sqrt{L}} \mathbf{s}_i^T \mathbf{Z}^L$$

is a zero-mean Gaussian random variable of variance $N_o/2$.

Let S be the $L \times L$ matrix whose i^{th} column is $\mathbf{s}_i$. The mutual orthogonality of the energy L sequences $\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_L$ implies that $S^T S = L\, I_L$, where I_L is the $L \times L$ identity matrix, and hence that $S^T = L S^{-1}$. Thus $SS^T = L\, I_L$ holds as well and hence we can recover $\mathbf{Y}^L$ from

$$\mathbf{R} = (R_1, R_2, \ldots, R_L) = \frac{1}{\sqrt{L}} S^T \mathbf{Y}^L$$

in the manner

$$\frac{1}{\sqrt{L}} S\, \mathbf{R} = \frac{1}{L} S\, S^T \mathbf{Y}^L = \mathbf{Y}^L. \tag{5}$$

The reversibility of this partial demodulator shows that it has discarded no information that was in the received vector $\mathbf{Y}^L$, cf. [3, p. 222]. But it follows also from (4) that the inputs B_j for $j \in G_i$ are independent of all demodulator outputs except R_i. Hence, the decoder for the i^{th} user group can now operate on the sequence of symbols from the single demodulator output labelled R_i to detect the users in this group only, without having to worry at all about the users in other groups and with no loss of optimality.

3 Coding for CDMA Systems with Partial Demodulation

We now consider coding for a CDMA system in which partial demodulation has been performed so that the demodulator output in any symbol period is given by (4) for the i^{th} user group. For specificity and simplicity, we assume that there are $m = 2$ users in each group. Without loss of essential generality, we consider coding for the first user group only, whose users we take to be users 1 and 2. We rewrite (4) to apply only to this user group in the manner

$$R = \sqrt{L}\,(B^{(1)} + B^{(2)}) + Z \tag{6}$$

where $B^{(1)}$ and $B^{(2)}$ are the modulation symbols of users 1 and 2, respectively, and where Z is a zero-mean Gaussian random variable with variance $N_o/2$. We further assume that the users employ binary antipodal modulation so that their modulation symbols takes values in the set $\{+\sqrt{E_c}, -\sqrt{E_c}\}$where E_c is the average chip energy. It is then natural to rewrite (6) for the i^{th} symbol at the demodulator output as

$$R_i = D_i^{(1)} + D_i^{(2)} + Z_i \qquad (7)$$

where $D_i^{(1)} = \sqrt{L}\, B_i^{(1)}$ and $D_i^{(2)} = \sqrt{L}\, B_i^{(2)}$ take values in $\{+\sqrt{E_s}, -\sqrt{E_s}\}$ and $E_s = LE_c$ is the average signal energy in one modulation symbol interval, and where Z_i is one in a sequence of independent zero-mean Gaussian random variables with variance $N_o/2$. Hereafter, we will forget chip level quantities entirely and consider $D_i^{(1)}$ and $D_i^{(2)}$ themselves to be the modulator output symbols for users 1 and 2, respectively.

The two-user channel described by (7) is familiar in multi-user information theory, cf. [4], where it is often called the noisy two-user *binary adder channel* (BAC). [It is customary in information theory to take each user's input alphabet to be {0, +1} rather than $\{+\sqrt{E_s}, -\sqrt{E_s}\}$ as is natural here, but this is a trivial difference that alters none of the theory of this channel.] This channel, together with its capacity region is shown in Fig. 3 for the *noiseless* case, i. e., for $N_o = 0$. The capacity region determines where reliable operation is possible in the sense that users 1 and 2 can, by proper choice of a coding technique, transmit information arbitrarily reliably (i. e., with error probability less than any specified positive number) through this channel at rates $r^{(1)}$ and $r^{(2)}$ (measured in information bits per channel use), respectively, if the rate pair $(r^{(1)}, r^{(2)})$ lies inside the capacity region, and cannot do this if $(r^{(1)}, r^{(2)})$ lies outside the capacity region. In the noisy case, i. e., when $N_o > 0$, the capacity region still has the pentagonal shape shown in Fig. 3, but is of course smaller.

4 Coding for the Binary Adder Channel

In the previous section, we reduced the coding problem for an idealized CDMA channel to the coding problem for the conceptually simply binary adder channel. We will begin our coding considerations for the BAC with the noiseless case where it is only the inter-user interference that reduces the capacity region from what it would be if each user enjoyed his own noiseless binary input channel; this reduction is reflected in the condition $r^{(1)} + r^{(2)} \leq 3/2$ bits/use, which corresponds to the 45^o line on the boundary of the capacity region in Fig. 3.

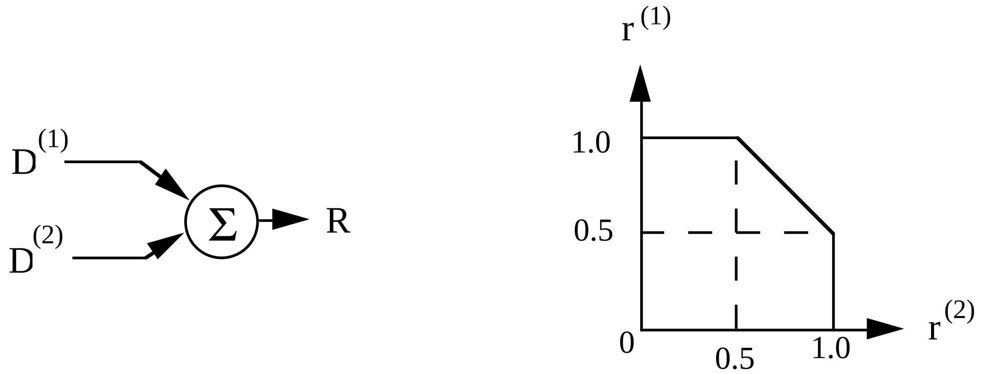

Fig. 3: The noiseless two-user binary adder channel and its capacity region. ($D^{(1)}$ and $D^{(2)}$ take values in $\{+\sqrt{E_s}, -\sqrt{E_s}\}$.)

Because the channel input alphabet $\{+\sqrt{E_s}, -\sqrt{E_s}\}$ or *signal set* for each user has two elements, the coding alphabet should also be binary. It is natural to take this latter alphabet to be the finite field GF(2) and to suppose that each user's modulator implements the mapping

$\mu(.)$ from GF(2) to the signal set given by $\mu(0) = +\sqrt{E_s}$ and $\mu(1) = -\sqrt{E_s}$. We now assume that both users employ a block code of blocklength n. For a codeword $\mathbf{b} = [b_1\ b_2\ \dots\ b_n]$ in such a code, we write $\mu(\mathbf{b})$ to denote the corresponding signal space sequence $(\mu(b_1), \mu(b_2), \dots \mu(b_n))$ that will be transmitted over the BAC when the encoder output is $\mathbf{b}$. It follows from (7) that the channel output sequence $\mathbf{R} = (R_1, R_2, \dots R_n)$ is given by

$$\mathbf{R} = \mu(\mathbf{b}^{(1)}) + \mu(\mathbf{b}^{(2)}) + \mathbf{N} \tag{8}$$

where $\mathbf{b}^{(1)}$ and $\mathbf{b}^{(2)}$ are the outputs of the encoders for users 1 and 2, respectively.

Example 1: Suppose that users 1 and 2 employ the blocklength n = 2 codes {[0 0], [1 1]} and {[0 1], [1 0]}, respectively, to encode one information bit each so that $r^{(1)} = r^{(2)} = 1/2$. Note from Fig. 3 that the rate pair $(r^{(1)}, r^{(2)}) = (1/2, 1/2)$ is well inside the capacity region of the noiseless BAC. The possible transmitted codewords, corresponding transmitted signals, and corresponding sum of transmitted signals are given in the following table:

$\mathbf{b}^{(1)}$	$\mathbf{b}^{(2)}$	$\mu(\mathbf{b}^{(1)})$	$\mu(\mathbf{b}^{(2)})$	$\mu(\mathbf{b}^{(1)}) + \mu(\mathbf{b}^{(2)})$
[0 0]	[0 1]	$\sqrt{E_s}\ (+1, +1)$	$\sqrt{E_s}\ (+1, -1)$	$\sqrt{E_s}\ (+2, 0)$
[1 1]	[0 1]	$\sqrt{E_s}\ (-1, -1)$	$\sqrt{E_s}\ (+1, -1)$	$\sqrt{E_s}\ (0, -2)$
[0 0]	[1 0]	$\sqrt{E_s}\ (+1, +1)$	$\sqrt{E_s}\ (-1, +1)$	$\sqrt{E_s}\ (0, +2)$
[1 1]	[1 0]	$\sqrt{E_s}\ (-1, -1)$	$\sqrt{E_s}\ (-1, +1)$	$\sqrt{E_s}\ (-2, 0)$

Because all four possible codeword combinations result in a different transmitted sum signal, the code pair ({[0 0], [1 1]}, {[0 1], [1 0]}) is said to be *uniquely decodable*, which means that the decoding error probability will be exactly 0 when the two users employ such a code pair on the noiseless BAC.

Much of the research on coding for the BAC, beginning with the seminal work of Kasami and Lin [4], has concentrated on the construction of uniquely decodable code pairs (or uniquely decodable T-tuples of codes for the T-user BAC). Because of the enormous amount of available theory and constructions for linear codes (i.e., codes that form a vector space over the field GF(2)), one would very much like to choose the component codes as linear codes if good code pairs of this type can be found. For this reason, the following result, which is due essentially to Peterson and Costello [5], has put a damper on the enthusiasm of coding theorists to work on codes for the BAC.

Proposition: If the components codes of a code pair (V_1, V_2) are binary linear codes of blocklength n whose rates satisfy $r^{(1)} + r^{(2)} > 1$, then this code pair is *not* uniquely decodable.

The proof is very simple. The dimensions k_1 and k_2 of V_1 and V_2 are $k_1 = nr^{(1)}$ and $k_2 = nr^{(2)}$, respectively. By one of the most fundamental results of linear algebra, cf. [6, Theorem 5.8],

$$\dim(V_1 \cap V_2) = \dim(V_1) + \dim(V_2) - \dim(V_1 + V_2)$$

where dim(.) denotes dimension and where $V_1 + V_2$ is the vector space consisting of all vectors that can be written as the sum (component by component in GF(2)) of a vector from V_1 and a vector from V_2. But surely $\dim(V_1 + V_2) \leq n$ so that $r^{(1)} + r^{(2)} > 1$ implies that

$$\dim(V_1 \cap V_2) \geq nr^{(1)} + nr^{(2)} - n > 0.$$

It follows that the codes V_1 and V_2 have at least one n*on-zero* codeword **b*** in common in addition to having the zero codeword **0** in common. But then $\mathbf{b}^{(1)} = \mathbf{b}^*$ and $\mathbf{b}^{(2)} = \mathbf{0}$ give the same sum of transmitted signals as does $\mathbf{b}^{(1)} = \mathbf{0}$ and $\mathbf{b}^{(2)} = \mathbf{b}^*$ so that the code pair is not uniquely decodable.

The undeniable truth of this proposition should not lead us to draw the wrong conclusion about the utility of linear coding for the BAC. It says only that we should not use binary linear codes to choose the binary antipodal signals, $+\sqrt{E_s}$, and $-\sqrt{E_s}$, that are sent directly over the BAC. One has other options as the following example shows.

Example 2: Let $a^{(1)}$ and $a^{(2)}$ be the informations bits of users 1 and 2, respectively, for the codes of Example 1 and let the encoding rules for the two users be $\mathbf{b}^{(1)} = [a^{(1)}\ \ a^{(1)}]$ and $\mathbf{b}^{(2)} = [a^{(2)}\ \ a^{(2)}+1]$ where the indicated sum is in GF(2). Suppose also that the demodulator output is further processed to deliver $Y^{(1)}$ and $Y^{(2)}$ where

$$Y^{(1)} = \frac{1}{\sqrt{2}}(R_1 + R_2)$$
$$Y^{(2)} = \frac{1}{\sqrt{2}}(R_1 - R_2). \qquad (9)$$

From the table in Example 1, we now deduce the following table for the noisy BAC:

$a^{(1)}$	$a^{(2)}$	$Y^{(1)}$	$Y^{(2)}$
0	0	$+\sqrt{2E_s} + N^{(1)}$	$+\sqrt{2E_s} + N^{(2)}$
1	0	$-\sqrt{2E_s} + N^{(1)}$	$+\sqrt{2E_s} + N^{(2)}$
0	1	$+\sqrt{2E_s} + N^{(1)}$	$-\sqrt{2E_s} + N^{(2)}$
1	1	$-\sqrt{2E_s} + N^{(1)}$	$-\sqrt{2E_s} + N^{(2)}$

where

$$N^{(1)} = \frac{1}{\sqrt{2}}(N_1 + N_2\}$$
$$N^{(2)} = \frac{1}{\sqrt{2}}(N_1 - N_2). \qquad (10)$$

It follows that $N^{(1)}$ and $N^{(2)}$ are zero-mean Gaussian random variables with mean 0 and variance $N_o/2$ and, moreover, are independent because $E[N^{(1)} N^{(2)}] = 0$. We see from this table that if one considers $a^{(i)}$ to be the modulator input and $Y^{(i)}$ to be the channel output of a *virtual channel* for user i, then we have decomposed two uses of the noisy two-user BAC, where each user sends binary antipodal signals of energy E_S on each use, into the situation where each user makes one use of his own virtual channel, which is a *white Gaussian noise channel with the same noise variance $N_o/2$ and with binary antipodal input signals of energy $2E_S$.* The energy efficiency for each user is just the same as if he were the only user of the original BAC. But the rate sum $r^{(1)}+ r^{(2)} = 1$ bits/use is well less than the 3/2 bits/use rate sum that the capacity region of Fig. 3 assures us can be approached for arbitrarily realiable transmission in the noiseless case.

The reader will surely have noticed that the complete and lossless separation of the users that was possible in Example 2 was the result of the fact that the the codewords $\mu(\mathbf{b}^{(1)})$ and $\mu(\mathbf{b}^{(2)})$ are always orthogonal, as can be seen from the table in Example 1. This "perfect" mutual orthogonality of the transmitted signals for the two users is possible only if the rate sum $r^{(1)}+ r^{(2)} \leq 1$ bits/use. Nonetheless, this example teaches what seems to us to be the major lesson to be learned about "coding" for the noisy BAC:

> *Simple uniquely decodable block codes for the noiseless BAC should be considered not as* ***codes*** *but rather as* ***signal sets*** *for use with a coding alphabet of the same cardinality. These signal sets should be chosen to make it possible to map the received word linearly to one (or at most a few) dimension(s) for each user in such a way that the resulting virtual channels, one for each user, exhibit substantial independence.*

Only in this way does it seem possible to avoid the prohibitive complexity associated with joint decoding of several users. A final example should make this lesson clearer.

Example 3: We now add a third codeword [0 0] to the code of user 2 in Example 1. The code pair is now ({[0 0], [1 1]}, {[0 0], [0 1], [1 0]}). The possible transmitted codewords and their sum are given in the following table:

$\mathbf{b}^{(1)}$	$\mathbf{b}^{(2)}$	$\mu(\mathbf{b}^{(1)})$	$\mu(\mathbf{b}^{(2)})$	$\mu(\mathbf{b}^{(1)}) + \mu(\mathbf{b}^{(2)})$
[0 0]	[0 0]	$\sqrt{E_S}(+1, +1)$	$\sqrt{E_S}(+1, +1)$	$\sqrt{E_S}(+2, +2)$
[1 1]	[0 0]	$\sqrt{E_S}(-1, -1)$	$\sqrt{E_S}(+1, +1)$	$\sqrt{E_S}(0, 0)$
[0 0]	[0 1]	$\sqrt{E_S}(+1, +1)$	$\sqrt{E_S}(+1, -1)$	$\sqrt{E_S}(+2, 0)$
[1 1]	[0 1]	$\sqrt{E_S}(-1, -1)$	$\sqrt{E_S}(+1, -1)$	$\sqrt{E_S}(0, -2)$
[0 0]	[1 0]	$\sqrt{E_S}(+1, +1)$	$\sqrt{E_S}(-1, +1)$	$\sqrt{E_S}(0, +2)$
[1 1]	[1 0]	$\sqrt{E_S}(-1, -1)$	$\sqrt{E_S}(-1, +1)$	$\sqrt{E_S}(-2, 0)$

Again we see that this code pair is uniquely decodable (and in fact this is the simple uniquely decodable code pair given by Kasami and Lin [4] as their "Example 1". Letting $a^{(2)}$, the information digit for user 2, be an element of the finite field GF(3) (as is required now since there are three codewords in user 2's code), choosing the encoding rule so that the codeword $\mathbf{b}^{(2)}$ is [0 0], [0 1] or [1 0] according as $a^{(2)}$ is 0, 1, or 2, respectively, and again defining $Y^{(1)}$ and $Y^{(2)}$ by (9), we obtain the following table:

$a^{(1)}$	$a^{(2)}$	$Y^{(1)}$	$Y^{(2)}$
0	0	$2\sqrt{2E_s} + N^{(1)}$	$0 + N^{(2)}$
1	0	$0 + N^{(1)}$	$0 + N^{(2)}$
0	1	$+\sqrt{2E_s} + N^{(1)}$	$+\sqrt{2E_s} + N^{(2)}$
1	1	$-\sqrt{2E_s} + N^{(1)}$	$+\sqrt{2E_s} + N^{(2)}$
0	2	$+\sqrt{2E_s} + N^{(1)}$	$-\sqrt{2E_s} + N^{(2)}$
1	2	$-\sqrt{2E_s} + N^{(1)}$	$-\sqrt{2E_s} + N^{(2)}$

We now see that user 2's virtual channel is a ternary Gaussian channel with the signal set $\{+\sqrt{2E_S}, 0, -\sqrt{2E_S}\}$ and that this channel is not influenced by the actions of user 1. User 1's virtual channel is indeed influenced by the actions of user 2, but there are two options open to overcome this. Either user 1's decoder can make use of the decoded ouput for user 2 to subtract the effect of that user from his signal or else it can decode as if both $\sqrt{2E_S}$ and $2\sqrt{2E_S}$ are signal points representing "0" for user 1 and as if both 0 and $-\sqrt{2E_S}$ are signal points representing "1" for user 1. The former option converts user 1's virtual channel to the same binary Gaussian channel that he enjoyed in Example 2 provided that user 2's decoder makes no error (and one would design the system so that this error probability was acceptably small); the latter option causes a loss of somewhat less than 6 dB for user 1 compared to what he enjoyed in Example 2 because the minimum Euclidean distance between signal points representing different values of $a^{(1)}$ has been reduced by a factor of 2 (but some signal points are still at the same Euclidean distance from the nearest signal point representing the opposite value of $a^{(1)}$ so that the net loss is somewhat less than 6 dB). This second option, while substantially less efficient than the first, has the advantage that the decoder for user 1 need pay no attention to the decoder for user 1.

5 Concluding Remarks

We have attempted above to give the outlines of a new approach to coding for multiple-access channels in general and CDMA channels in particular. The key ideas are, first, to perform partial demodulation that segregates the users into small groups that can be processed independently with little or no loss and, second, to devise "small codes" or "signal sets" for each user group that allow independent decoding for each user (possibly with cancellation to remove the effect of signals of the various users as they are decoded) with little or no additional loss. This "signal-set" design is much more a task for the modulation engineer than for the coding theorist, as one cannot expect to obtain elegant algebraic solutions of the signal-set design problem. Moreover, this task will have to take into account many of the real-world complications of the problem, which we ignored, such as lack of symbol and/or chip synchronization, different user energies, time variations of the channel, multipath effects, and the like. The task is not an easy one, but it seems to us worth doing.

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