Spectrum Spreading and Multiple Accessing

James L. Massey
Signal & Info. Proc. Lab., Swiss Federal Inst. Tech., CH-8092 Zurich, Switzerland

Abstract — **The novel definition of a spread–spectrum signal as a modulated signal whose Fourier bandwidth is much greater than its Shannon bandwidth (i.e., one-half the minimum number of dimensions per second required for a signal space representation of the signal) is used as the starting point in an investigation of the connection between spectrum–spreading and multiple–accessing. This conceptual framework admits a very natural treatment of the condition for no interuser interference in a multiple–access system. Examples are offered to make plausible the thesis that this conceptual framework is also useful in understanding and comparing practical random–access systems in which the "ideal" of no interuser interference is impossible or infeasible to achieve.**

I. Introduction

The purpose of this paper is to explore the connection between spectrum spreading and multiple accessing. This task requires at the outset that we have a clear notion of what one means by "spectrum spreading". In [1], we defined a *spread-spectrum signal* as a modulated signal whose Fourier bandwidth is much greater than its shannon bandwidth—we will abide here by this succint, but somewhat controversial, characterization. By the *Shannon bandwidth*, B, of a modulated signal, we mean one–half the minimum number of orthonormal functions per second that are required in a basis for a signal space in which this signal can be represented. Equivalently, the Shannon bandwidth is one-half the minimum number of dimensions per second required for a signal space representation of the signal. Because at most $2W$ dimensions per second can be accomodated in a Fourier bandwidth of W Hz, it follows that the Fourier bandwidth, W, and the Shannon bandwidth, B, must satisfy

$$B \leq W \ .$$

[Near] equality holds in this inequality when the orthonomal functions are [approximately] the "sinc pulses" $\sqrt{2W}\mathrm{sinc}(2Wt - n)$ for all integers n, where $\mathrm{sinc}(x) = \sin(\pi x)/(\pi x)$. It is thus natural, as in [1], to define the *spreading factor*, γ, of a modulated signal by

$$\gamma = W/B \ .$$

A modulated signal is then a spread–spectrum signal when its spreading factor is large, say $\gamma \geq 5$. Note that the representation of a signal with spreading factor γ requires $2W/\gamma$ orthonormal functions per second in the basis for the signal space.

For simplicity and convenience, we will consider only baseband signals in this paper, but all the results carry over easily to the bandpass signal case.

II. Condition for Zero Interuser Interference

We will confine our discussion of multiple accessing to the situation where K users send their modulated signals to a common receiver. We assume that the received signal is the sum of the appropriately scaled and filtered signals from each user, which we shall refer to as the signals *as seen at the receiver*, and additive white Gaussian noise. We also assume that the modulated signal of each user, as seen at the receiver, is a baseband signal of Fourier bandwidth W. That is, all K signals have the same Fourier bandwidth and occupy the same part of the spectrum—we will refer to this assumption as the *common Fourier bandwidth* assumption. Note that this assumption excludes frequency–division multiple–access (FDMA) systems from our discussion, but includes their "dual", namely time–division multiple–access (TDMA) systems.

The following assertion is both trivial and insightful.

Proposition 1 *Suppose that K users send their modulated signals to a single receiver, using a common Fourier bandwidth of W Hz. Then zero interuser interference (IUI) is possible at the receiver only if the users transmit spread–spectrum signals whose spreading factors satisfy*

$$\sum_{i=1}^{K} 1/\gamma_i \leq 1 \ .$$

This result follows from the fact that there will be no IUI if and only if the K received signals can be individually represented in K pairwise–orthogonal subspaces of the signal space. For this to be the case, the number of dimensions per second of the signal space required to represent the received signal must be at least the sum of the number of dimensions per second, $2W/\gamma_i$, required for representing user i's signal for $i = 1, 2, \ldots K$. But this total number also cannot exceed $2W$ since the sum signal lies in a Fourier bandwidth of W Hz.

The previous proposition admits a kind of converse, hardly less trivial than the proposition itself.

Proposition 2 *Suppose that K users send their modulated signals to a single receiver, using a common Fourier bandwidth of W Hz. Then zero interuser interference (IUI) is achieved at the upper limit of the sum of the reciprocal spreading factors*

$$\sum_{i=1}^{K} 1/\gamma_i = 1$$

when (1) the access system is a fully synchronized TDMA system in which each user's spreading factor is $\gamma = K$, or when (2) the access system is a fully synchronized CDMA system in which the symbol signature sequences of the K users are sequences of $+1$'s and -1's that form the rows of a $K \times K$ Hadamard matrix and the corresponding "chip" waveforms are sinc pulses of Fourier bandwidth W Hz.

Our interest in the above two rather trivial propositions stems from the fact that they illustrate that the notions of a spread-spectrum signal and its spreading factor, as given in [1], provide a very natural framework for discussing the complete absence of interuser interference in interesting kinds of multiple–access systems.

III. More Practical Considerations

In many practical multiple–access systems, particularly those that arise in mobile radio communications, it is impossible to achieve zero interuser interference because of the infeasibility of fully synchronizing the users' signals and of fully compensating for the distortion of the transmitted signals due to multipath propagation, and other practical considerations. The thesis of this paper is that the notions of a spread-spectrum signal and its spreading factor as given in [1] are also useful in understanding and comparing multiple–access systems in such practical settings.

In the presentation of this paper, we will show that wideband (but not spread!) modulated signals, such as pulse-position modulation (PPM) signals with $\gamma = 1$, not only cannot provide zero IUI but, more fundamentally, are inherently inferior to spread signals, such as TDMA signals, for common Fourier bandwidth multiple–accessing. This demonstration will be made be comparing the capacity regions of the discrete memoryless multiple–access channels corresponding to each choice of signals. Other examples will be used to illustrate the fact that two multiple–access systems, both of which deliver zero IUI in the fully synchronized case and both employing spread signals all having the spreading factor $\gamma = K$, can differ widely in their performance in the partially synchronized and unsynchronized cases. The *degeneracy* of the modulated signals, as measured by collapses in the dimensionality of the received signal with losses in synchronization, will be seen to play an important role in determining the deterioration of performance.

It was shown in [1] that (nontrivial) coding generally increases both the Shannon bandwidth and the Fourier bandwidth of a signal by the same amount, i.e., it does *not* spread the signal. The presentation of this paper will conclude with an exploration of the question of whether, as a result of this property of coding, it is better in a multiple–access system to widen the bandwidth of the transmitted signals by coding alone, by spreading alone, or partly by coding and partly by spreading.

References

[1] J. L. Massey, "Towards an Information Theory of Spread–Spectrum Systems" in *Code Division Multiple Access Communications* (Eds. S. G. Glisic and P. A. Leppänen). Boston, Dordrecht and London: Kluwer, 1995, pp. 29–46.