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An Error Bound for Random Tree Codes

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Abstract

An \((n, k, L, M)\) binary tree code is defined as the assignment of \(n\) binary digits to each of the branches in a rooted tree for which the number of branches diverging from each node at depth \(i\) from the root is \(2^i\) for \(i < L\) and is \(1\) for \(L \leq i < L + M\). The constraint length of such a code is defined as \(N = (M+1)n\) which is the length of the "long branches" diverging from the nodes at depth \(L-1\) from the root, and the rate is defined to be \(R = k/n\).

(This corresponds to the usual definitions of constraint length and rate for convolutional codes which are a special subclass of linear \((n, k, L, M)\) tree codes.) It is shown that the average probability of decoding error with maximum likelihood decoding on a discrete memoryless channel for the ensemble of \((n, k, L, M)\) tree codes satisfies

\[
\overline{P(\mathcal{E})} \leq \exp \left[ NE_v(R) \right] \text{ independent of } L
\]

where \(E_v(R)\) is the error exponent found by Viterbi for the ensemble of time-varying convolutional codes. The new feature of this bound is the lack of dependence on the tree length; some practical consequences of this independence are pointed out.

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