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An Error Bound for Random Tree Codes [1]

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Abstract

An (n,k,L,M) binary tree code is defined as the assignment of n binary digits to each of the branches in a rooted tree for which the number of branches diverging from each node at depth i from the root is 2^k for $i < L$ and is 1 for $L \leq i < L + M$. The constraint length of such a code is defined as $N = (M+1)n$ which is the length of the "long branches" diverging from the nodes at depth L-1 from the root, and the rate is defined to be $R = k/n$. (This corresponds to the usual definitions of constraint length and rate for convolutional codes which are a special subclass of linear (n,k,L,M) tree codes.) It is shown that the average probability of decoding error with maximum likelihood decoding on a discrete memoryless channel for the ensemble of (n,k,L,M) tree codes satisfies

$$\overline{P(E)} \leq \exp\ [NE_v(R)] \text{ independent of } L$$

where $E_v(R)$ is the error exponent found by Viterbi for the ensemble of time-varying convolutional codes. The new feature of this bound is the lack of dependence on the tree length; some practical consequences of this independence are pointed out.

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