A LOWER BOUND ON THE MINIMUM DISTANCE GROWTH RATE OF FIXED CONVOLUTIONAL CODES*

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SUMMARY

Massey and Sain [1] have defined a convolutional code as subject to catastrophic error propagation if a finite number of transmission errors can cause an infinite number of errors in decoding the information sequence. This can happen if an information sequence of infinite weight generates a codeword of finite weight. In terms of the modified encoder state diagram, this implies a cycle of zero weight.

In practical terms, catastrophic codes are subject to high bit error probabilities when used with maximum likelihood or sequential decoding. This follows from the fact that the zero weight loop can be used to generate long codewords with low weight which contain a large number of non-zero information bits. A small number of channel errors can then cause the decoder to estimate the all-zero codeword, and hence the all-zero information sequence, thereby leading to a large number of information bit decoding errors.

In a similar vein, if an encoder is non-catastrophic, but contains a loop with a low fractional weight (weight per branch), it can produce long codewords with low weight, leading to high decoded bit error rates. In order to determine whether a convolutional code can contain long codewords with low weight, the rate of growth of the minimum distance between unmerged codewords must be investigated. In an earlier paper, Hemmati and Costello [2] proved that the rate of growth of the minimum distance between unmerged codewords in the ensemble of rate R = K/N

time-varying convolutional codes meets the Gilbert bound, i.e.,

$$D/NT \geq H^{-1}(1-R), \qquad (1)$$

where D is the minimum distance between unmerged codewords and T is the length of the information sequence (in K-bit blocks).

In this paper we establish a similar result for the class of fixed (time-invariant) convolutional codes. This new result is based on a bound on definite decoding minimum distance originally obtained by Massey [3] and later improved by Miczo [4]. In particular, we have shown that there exist fixed convolutional codes for which the rate of growth of the minimum distance between unmerged codewords is bounded by

$$D/NT \geq [(1+R)/3]\, H^{-1}[(1/6)[(1-R)/(1+R)]]. \qquad (2)$$

This result establishes the existence of fixed convolutional codes, of any constraint length, for which the minimum distance between unmerged codewords grows linearly as a function of codeword length. This guarantees that long codewords cannot have low weight, and hence that decoder error events are short and contain only a relatively few information bit errors.

References:

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