(Submitted as short paper to ISIT-85)

30.11.84

ON THE MUTUAL INFORMATION AND CUT-OFF RATE OF CHANNELS WITH INTERSYMBOL INTERFERENCE

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A B S T R A C T

The channels studied are discrete-time channels in which the channel input filter has a unit-sample response with a memory of M past samples, and in which a white Gaussian noise sequence is added to the filter output. The per digit mutual information I and the cut-off rate R_0 are studied for i.i.d. input sequences. Two cases are considered: Gaussian input samples and equally-likely binary antipodal input samples.

For the Gaussian input case, similar and simple expressions for I and R_0 are obtained. These expressions show that I coincides with R_0 when the latter is evaluated for twice the signal energy -- a result long known to hold in the memoryless case (M = 0) where I is capacity.

For the binary input case, R_0 is found (by an adaptation of a Method due to Omura) as the negative logarithm of the maximum eigenvalue of a certain state transition matrix. For given small M, worst-case-R_0 channels are identified and are shown to change with signal-to-noise ratio. Upper and lower bounds on I are derived, and evaluated numerically for certain interesting channels.

The above results are interpreted in terms of the loss in mutual information and/or capacity that results from intersymbol interference. One as yet unexplained result is that, for certain channels, the binary input R_0 exceeds the Gaussian input R_0.

S U M M A R Y

The channel model is a finite-memory discrete-time channel such that the channel output at time i is $y_i = h_0 x_i + h_1 x_{i-1} + \ldots + h_M x_{i-M} + z_i$ where x_i and z_i are the input and additive noise, respectively, at time i and where $[h_0, h_1, \ldots, h_M]$ is the unit-sample response of the channel. The additive noise sequence is i.i.d. and Gaussian with mean 0 and variance $N_0/2$. The unit-sample response is normalized as $h_0^2 + h_1^2 + \ldots + h_M^2 = 1$. The channel input constraint is $E[x_i^2] \leqslant \mathcal{E}$ for all i. The quantities of interest are the per digit mutual information $I = \lim_{n\to\infty} I(x^n;\, y^n)/n$ and the cut-off rate R_0, both computed when the input sequence is i.i.d. with a specified probability distribution. Two cases are studied: The Gaussian input distribution with mean 0 and variance $\mathcal{E}$, and the binary input distribution in which $P(x=\sqrt{\mathcal{E}})=P(x=-\sqrt{\mathcal{E}}) = 1/2$.

For the Gaussian input distribution, it is shown that

$$I=\frac{1}{4\pi}\int_0^{2\pi} \log\left\{1+\frac{2\mathcal{E}}{N_0}\left[1+2\sum_{k=1}^{M}\sum_{i=k}^{M} h_i h_{i-k} \cos(k\lambda)\right]\right\}d\lambda.$$

It is shown further that R_0 is given by the same expression with $\mathcal{E}$ replaced by $\mathcal{E}/2$ -- this mirrors the well-known result for the memoryless case (M = 0) where I is capacity. The expression for I can be derived from Shannon's frequency-domain parallel-channel approach, but also by a direct time-domain approach which was employed to obtain the expression for R_0.

For the binary input distribution, a modification of an approach developed by Omura is used to obtain $R_0 = -\log \lambda$, where λ is the largest eigenvalue of a certain $[\frac{1}{2}(3^M-1) + 1] \times [\frac{1}{2}(3^m-1) + 1]$ channel state transition matrix. This approach permits the explicit identification of <u>worst-case</u>-R_0 <u>channels</u> for given small memory M. Unlike the previously studied worst-case

Euclidean distance channels, the worst-case-R_0 channels are shown to change with the signal-to-noise ratio E/N_0. Some curious examples will be given where the binary input R_0 exceeds the Gaussian input R_0 for the same channel -- no such inequalities have been observed for the binary input I and the Gaussian input I, however. Upper and lower bounds on the binary input I have been obtained as multi-dimensional integrals. These integrals have been evaluated by Monte Carlo methods for some interesting intersymbol interference channels. The numerical results will be given during the oral presentation.

The results of this work permit one to make some quantitative statements about the loss in mutual information and/or R_0 that results from intersymbol interference, as well as about the necessary additional complexity of coding systems.