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Node Synchronization of R=1/2 Binary Convolutional Codes

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ABSTRACT

Binary rate-1/2 convolutional encoders with code generating-polynomials $G_1(D)$ and $G_2(D)$ are considered where $G_i(0)=1$. The node synchronization problem is to determine the boundary between the length-two subblocks in the encoded sequence when the receiver enters this sequence at an arbitrary point and observes N consecutive encoded digits. For the noiseless case, it is shown that the probability of synchronization error depends only on the degree of the "unified code-generating polynomial", $G(D) = G_1(D^2) + D \cdot G_2(D^2)$, provided that the encoder is non-catastrophic. The received sequences that are ambiguous in the sense of not uniquely specifying node synchronization are shown in the non-catastrophic case to be the set of all length N output sequences that can be produced by a linear feedback shift-register with connection polynomial G(D). Simulation results for node synchronization error probability when the encoded sequences are transmitted over a binary symmetric channel are used to show that the synchronization performance in the noiseless case is a reliable predictor of performance in the noisy case as well.

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SUMMARY

Consider an R=1/2 binary convolutional encoder (with subblock lengths $n_0=2$) with code generating polynomials $G_1(D)$ and $G_2(D)$. Defining the unified code-generating polynomial

$$G(D) = G_1(D^2) + D \cdot G_2(D^2) ,$$

one can write the serialized encoded sequence as

$$T(D) = I(D^2) \cdot G(D) \qquad (1)$$

where $I(D) = \ldots + i_{-1}D^{-1} + i_0 + i_1 D + \ldots$

is the D-transform of the (two-sided) information sequence. We assume, without loss of essential generality, $G_1(0)=1$.

Now suppose that the receiver lacks "node synchronization" (i.e., it does not know the boundary between subblocks) and must enter the encoded stream with the encoder already under way -- as is often the case in practical applications of convolutional codes.

Then the transmitted sequence, relative to the receiver's time origin, could equally well appear as the sequence

$$\tilde{T}(D) = D \cdot \tilde{I}(D^2) \cdot G(D) \tag{2}$$

from the "out-of-phase encoder". The synchronization problem considered in this paper is for the receiver, from observation of the received sequence over N consecutive time instants, to decide whether this sequence came from the "in-phase encoder" described by (1) or from the "out-of-phase encoder" described by (2) when both alternatives are equally likely and when the information sequence is the output of a binary symmetric source. Let $P_N(\varepsilon)$ denote the probability of error in this synchronization decision.

The noiseless case where the received sequence R(D) is either T(D) or $\tilde{T}(D)$ is considered first. It will be shown that, provided that the encoder is not catastrophic (i.e., provided that $\gcd[\, G_1(D), G_2(D)\,] = 1$),

$$P_N(\varepsilon) = \begin{cases} 1/2 & N \le L \\ (1/2)^{(N-L+3)/2} & N > L\,, \quad N-L \text{ odd} \\ (1/2)^{(N-L)/2+1} & N > L\,, \quad N-L \text{ even} \end{cases} \tag{3}$$

for the optimum (i.e. maximum-likelihood) synchronization rule, where L is the degree of G(D). [Note that the encoder $\{G_1(D)=1, G_2(D)=1+D\}$ with $G(D)=1+D+D^3$ and hence L=3 performs worse than the encoder $\{G_1(D)=1+D, G_2(D)=1\}$ with $G(D)=1+D+D^2$ and L=2, although these encoders differ only in the order in which the $n_0=2$ encoded sub-sequences are serialized.] It will further be shown that when the encoder is catastrophic, then the maximum-likelihood (ML) synchronization rule gives the same $P_N(\varepsilon)$ as for the non-catastrophic encoder obtained by dividing $G_1(D)$ and $G_2(D)$ by their greatest common divisor -- because this division reduces L, it follows that catastrophic encoders in a sense give a superior noiseless synchronization performance compared to the non-catastrophic encoders.

Let $\{R(D)\}_0^N = r_0 + r_1 D + \ldots + r_{N-1} D^{N-1}$ denote the received sequence over time instants 0,1,...,N-1. We say in the noiseless case, that $\{R(D)\}_0^N$ is ambiguous if it does not permit perfect synchronization, i.e., if there exists $I(D)$ and $\tilde{I}(D)$ such that $\{R(D)\}_0^N = \{I(D^2) \cdot G(D)\}_0^N = \{D \cdot \tilde{I}(D^2) \cdot G(D)\}_0^N$. Note that this implies

$$\{[I(D^2) + D \cdot \tilde{I}(D^2)] G(D)\}_0^N = 0$$

or, equivalently, that $I^*(D) = I(D^2) + D \cdot \tilde{I}(D^2)$, which can be an arbitrary power series, satisfies

$$\{I^*(D) \cdot G(D)\}_0^N = 0 \ . \qquad (4)$$

But equation (4) is equivalent to the condition that $\{I^*(D)\}_0^N$ be a length N output sequence from the length L binary linear feedback shift-register (LFSR) with connection polynomial G(D). When G(D) is primitive (which happens for surprisingly many of the best short-constraint-length codes used in practice), there are only two essentially different choices for $\{I^*(D)\}_0^N$, namely the all-zero sequence and the maximum-length (ML) sequence of period 2^L-1 (which of course has 2^L-1 different "phases" depending on where one enters this sequence.) By the shift-and-add property of ML sequences, it follows that these same 2 sequences are the only essentially different choices for an ambiguous $\{R(D)\}_0^N$. In the general non-catastrophic case, the set ambiguous $\{R(D)\}_0^N$ is also just the set of length N output sequences from the LFSR with connection polynomial G(D).

The ML synchronization rule will be derived for the case when the encoded sequences are sent over a binary symmetric channel with cross-over probability p, $p<1/2$. Simulation results for various encoders show that the achieved $P_N(\varepsilon)$ for the noisy case is ordered for different encoders in the same manner as for the noiseless case. The noisy case is currently being investigated and it is expected that, by the time of the Symposium, analytical results for the noisy case will be available and can be presented.