Low-Pass Filter Effect in the Measurement of Surface EMG

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Abstract

Many publications and articles describe the electrical properties of muscular tissue. In these articles the terms "low-pass filter" and "purely resistive medium" often occur in parallel. From the viewpoint of electrical engineering (namely from a network theoretic viewpoint) one would not expect that a purely resistive medium behaves as a low-pass filter. Instead the existence of some capacitive and/or inductive elements would be expected.

In this paper we present a network theoretic description of the low-pass filter effect of muscular tissue. We will use a descretized and simplified tissue model, e.g. a network of resistors. For this model the low-pass characteristic of muscular tissue will be verified.

The fundamental reason for the low-pass characteristic of muscular tissue is the fact that electrical activity travels with some constant velocity along a given path (e.g. the muscle fibres) while the measurement of this activity is done at some fixed point. The independant variables "time" and "space" can be mapped onto each other. The low-pass filter effect occurs in space domain.

1: Introduction

In the literature on the description of electrical activity within living organisms, one often encounters the terms "low-pass filter" and "purely resistive medium" ([8], [7], [2]). From the viewpoint of network theory it is known, that building any type of frequency selective filters (using continuous elements), capacitive and/or inductive elements are required. For the description of the low-pass filter effect of tissue, the fundamental differential equation, namely Maxwell's current law, must be solved. Due to the complicated formalisms, the fundamental reason for the low-pass filter effect of tissue remains somewhat hidden. Using a discretized model of the physiological processes that cause electrical activity within tissue, the nature for the low-pass filter effect of tissue can be better verified and clarified.

In this paper we start with a short physiological description of electrical activity in living organisms. As a simplified model we will then present a network of resistors which is driven by some moving current sources. This arrangement is sufficient for the description of the nature of the low-pass filter effect of tissue. The low-pass filter effect of tissue is then described using network theoretic terms.

2: Physiology

Electrical activity within tissue is caused due to the transport of positive and negative ions. This transport of ions is in effect when a nerve or a muscle fiber becomes active. The many biochemical processes that lead to this activity are not discussed here. Instead we assume some given distribution of current sources.

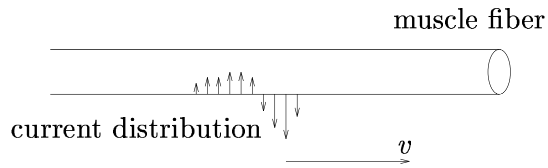

Figure 1. Electrical activity of a muscle fiber (schematically).

The electrical activity of a muscle fiber within tissue is schematically described in figure 1. The muscle fiber is a cylinder with a diameter of some 50μm and a length of some centimeters. The current distribution consists of a flow of positive and negative ions between the inner and outer part of the muscle fiber. In figure 1 the arrows indicate the resulting positive current flow. The current flow along the perimeter of the muscle fiber shall be constant. The net current flow in or out of the local region of this current distribution has to be 0. In other words, no current flows in or out of this local region. The current distribution travels with a constant velocity v along the muscle fiber.

3: Measuring the electrical activity

The current distribution is a continous distribution of current sources. The outer part of the muscle fiber (tissue) is a purely resistive medium. Depending on the conductivity of this tissue a certain current field will be established. The measurement of electrical activity within living organisms is performed by adding some kind of electrodes. In general, these electrodes disturb the current field. Below, we will assume an arrangement which is not disturbed.

4: A network of resistors

Figure 2 is a discretized description of above considerations. It is a snapshot for a given time point. The potentials $\phi_n(z)$ and $\phi_f(z)$ are functions of the location z. Due to the velocity v, an observation of the potential distribution for a given location z leads to time-dependant functions $\phi_n(z + vt)$ and $\phi_f(z + vt)$. This is the viewpoint, when the electrical activity within living organisms is measured. The space-dependant and the time-dependant descriptions can be mapped onto each other (constant velocity v). The description of the low-pass filter effect is performed using the space-dependant description.

We describe the contents of figure 2 in some more detail. The current sources modelize the current flow through the surface of the muscle fibre. The fact of a local current distribution is fulfilled (no current flows out of the local region). The tissue is considered to be a homogeneous, anisotropic medium. The resistive properties do not change along the z-axis and the anisotropy factor is λ^2. $\phi_n(z)$ is the near fiber potential distribution and $\phi_f(z)$ is the far fiber potential distribution. We assume a very thin piece of tissue. A one-layer network of resistors shall be sufficient for its description.

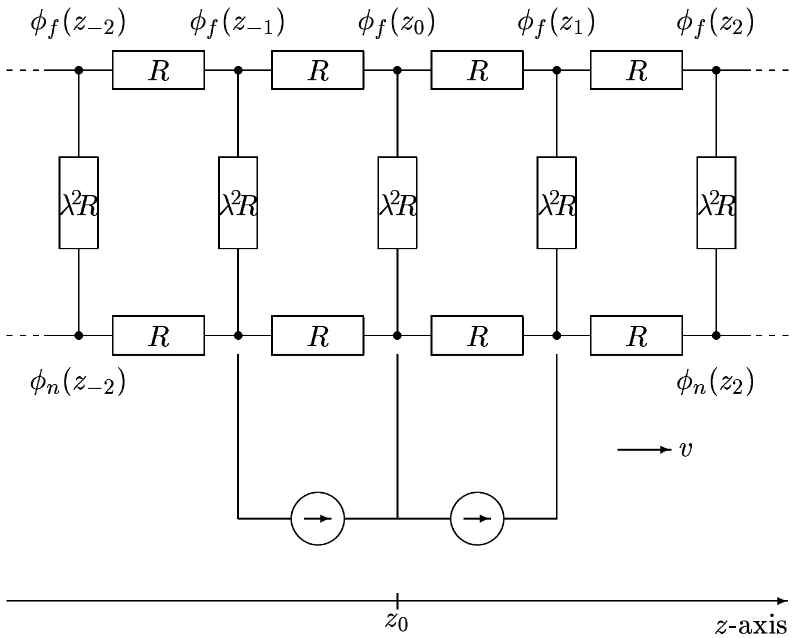

Figure 2. Discrete network of resistors

5: Existence of a transfer function

Using space-invariance and linearity we prove the existence of a transfer function. Let us consider a single current source located at $l = 0$ as depicted in figure 3. The condition of a local current distribution is fulfilled.

Choosing the value of the current source $I[0] = 1$ shall lead to the (unknown) potentials

$$\phi_n[.] = a[.] \quad (1)$$
$$\phi_f[.] = b[.]\ . \quad (2)$$

Moving the current source to a different location $l = l_1$ ($I[l_1] = 1$) will then lead to the potentials (space-invariance)

$$\phi_n[.] = a[. - l_1] \quad (3)$$
$$\phi_f[.] = b[. - l_1]\ . \quad (4)$$

Any arrangement of current sources $I[l] = c[l]$ leads to the potentials (linearity)

$$\phi_n[.] = \sum_l c[l] \cdot a[. - l] \quad (5)$$
$$\phi_f[.] = \sum_l c[l] \cdot b[. - l]\ . \quad (6)$$

The $\mathbf{z}$-transform of the potentials $\phi_n[.]$ and $\phi_f[.]$ are

$$\Phi_n(\mathbf{z}) = C(\mathbf{z}) \cdot A(\mathbf{z}) \quad (7)$$
$$\Phi_f(\mathbf{z}) = C(\mathbf{z}) \cdot B(\mathbf{z})\ . \quad (8)$$

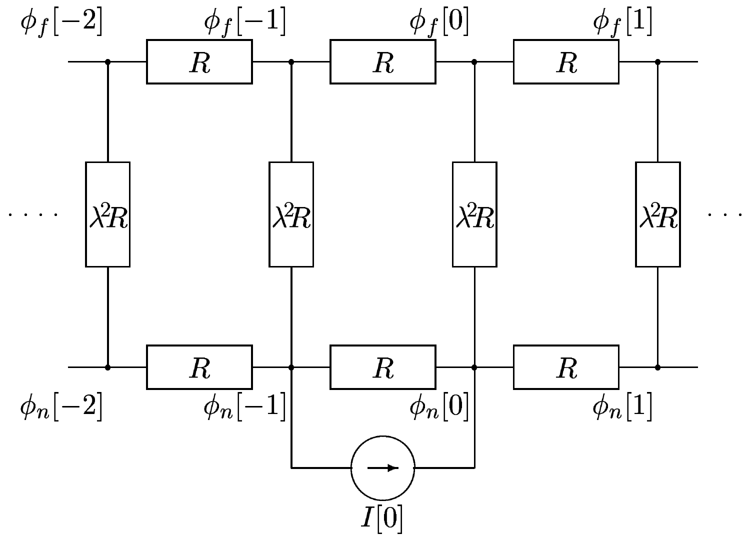

Figure 3. Discrete network of resistors with single current source

A transfer function exists and can be calculated using

$$H(\mathbf{z}) = \frac{\Phi_f(\mathbf{z})}{\Phi_n(\mathbf{z})} = \frac{B(\mathbf{z})}{A(\mathbf{z})} . \tag{9}$$

6: Calculation of the transfer function

We calculate the transfer function $H(\mathbf{z})$. We consider a network of resistors as shown in figure 4. We assume an arbitrary number of current sources $I[l]$. For the location l the mesh equations (compare with figure 4)

$$\phi_f[l] = Ri_f[l] + \phi_f[l+1] \tag{10}$$

$$\phi_n[l] = \lambda^2 Ri_q[l] + \phi_f[l] \tag{11}$$

must be fulfilled. The currents in figure 4 can therefore be expressed as

$$i_f[l] = \frac{1}{R}\left(\phi_f[l] - \phi_f[l+1]\right) \tag{12}$$

$$i_q[l] = \frac{1}{\lambda^2 R}\left(\phi_n[l] - \phi_f[l]\right) . \tag{13}$$

Further, these currents have to fulfill the knot equation (knot for the potential $\phi_f[l]$)

$$i_f[l-1] + i_q[l] = i_f[l] . \tag{14}$$

Inserting equations 12 and 13 into the equation 14 leads to

$$\lambda^2\left(\phi_f[l-1] - \phi_f[l]\right) + \phi_n[l] - \phi_f[l] = \lambda^2\left(\phi_f[l] - \phi_f[l+1]\right) \tag{15}$$

or

$$-\lambda^2\phi_f[l-1] + (1+2\lambda^2)\phi_f[l] - \lambda^2\phi_f[l+1] = \phi_n[l] . \tag{16}$$

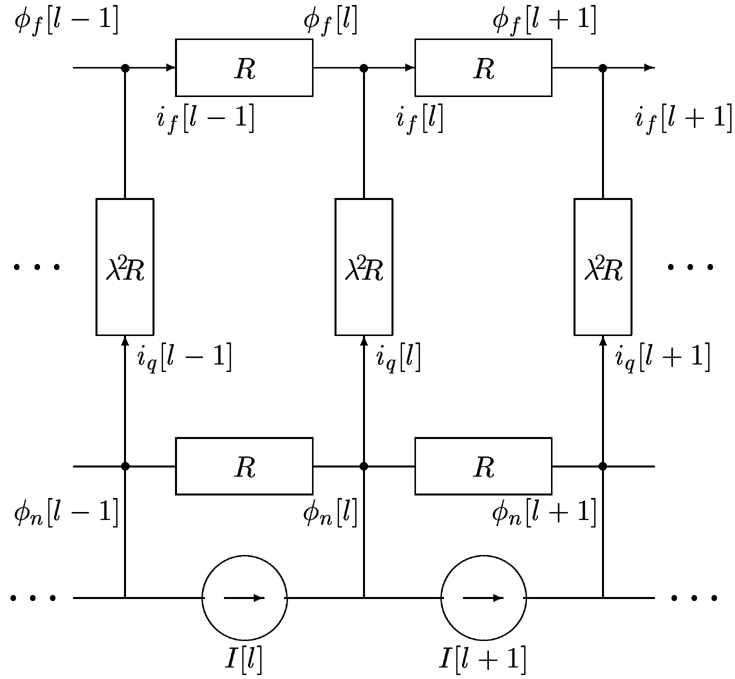

Figure 4. Discrete network of resistors with arbitrary current sources

Equation 16 can now be $\mathbf{z}$-transformed and then solved for the transfer function $H(\mathbf{z})$:

$$H(\mathbf{z}) = \frac{\Phi_f(\mathbf{z})}{\Phi_n(\mathbf{z})} = \frac{1}{-\lambda^2\mathbf{z}^{-1} + (1+2\lambda^2) - \lambda^2\mathbf{z}^{+1}} \,. \tag{17}$$

Choosing $\mathbf{z} = \exp j\omega$ leads to the frequency response

$$H(\omega) = \frac{1}{1 + 2\lambda^2\,(1-\cos\omega)} \,. \tag{18}$$

In figure 5 the frequency response $H(\omega)$ is drawn for different values of the anisotropy factor λ^2. The frequency response has always a low-pass characteristic.

7: Conclusions

A description of the low-pass filter effect of tissue has been presented from the viewpoint of network theory. This description gives a comprehensive insight to the fundamental reason of the low-pass filter effect. The two terms “low-pass filter” and “purely resistive medium” coexist well. The description of the “low-pass filter” effect is done using space-dependent functions. The resulting transfer function is non-causal. When dealing with space-dependent functions, non-causality does not harm realizability.

References

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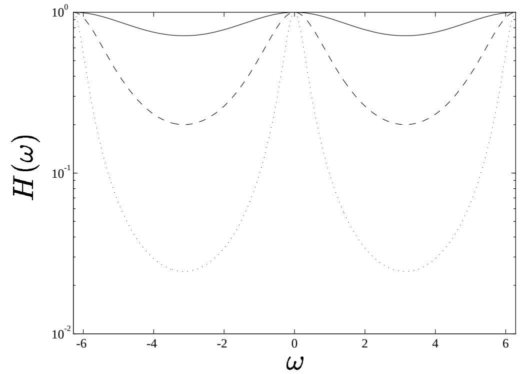

Legend: — $\lambda^2 = 1/5$, -- $\lambda^2 = 1$, ··· $\lambda^2 = 5$

Figure 5. Frequency response $H(\omega)$ for different anisotropy factors λ^2.

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