Analog Circuits for Symbol-Likelihood Computation

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***Abstract*— New analog circuits are presented to convert the output of a matched filter into symbol- and bit-likelihoods suitable as input to analog decoders for error correcting codes. The new circuits can be adapted to many constellations including pulse amplitude modulation (PAM) and quadrature amplitude modulation (QAM) and for arbitrary bit-to-symbol mappings. In contrast to a previously published circuit these new circuits compute exact probabilities and they are more versatile.**

I. INTRODUCTION

This paper addresses a problem that arises in the context of analog decoders for error correcting codes [1]–[4].

Consider the standard discrete-time model of the additive white Gaussian noise (AWGN) channel. Let X be the (real or complex) transmitted signal, which is taken from some finite constellation $\mathcal{A} = \{A_0, \ldots, A_{M-1}\}$. The received symbol Y is

$$Y = X + Z, \tag{1}$$

where Z is a zero-mean (real or complex) Gaussian random variable with variance σ_Z^2 that is independent of X. Let $f(y|x)$ be the conditional probability density of Y given $X = x$.

In this paper, we present novel analog circuits for the computation of $f(y|x)$. The input to the circuits is the (real or complex) number $Y = y$; the output of the circuits consists of currents proportional to the M real numbers

$$f(y|X = A_0), \ldots, f(y|X = A_{M-1})$$

in a form suitable for analog decoders. The circuits are easily extended to produce bit likelihoods ("soft bits") for arbitrary bit-to-symbol mappings.

For binary antipodal signaling with $\mathcal{A} = \{+1, -1\}$, it has long been known[1] that the circuit of Fig. 1 can be used to compute $f(y|x)$. The output of this circuit are the two currents I_+ and I_-, which are proportional to $f(y|+1)$ and $f(y|-1)$, respectively.

A circuit to compute a good approximation of $f(y|x)$ for a square QAM constellation was proposed by Seguin et al. [7].

In this paper, the circuit of Fig. 1 is generalized to arbitrary constellations and bit-to-symbol mappings. In contrast to the circuits of [7], the new circuits of this paper compute $f(y|x)$ exactly (up to the validity of the exponential transistor model) and they are more versatile.

[1]The circuit was mentioned in [5], but was known earlier. A good description may be found in the appendix of [6].

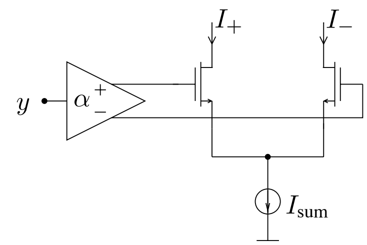

Fig. 1. Well known circuit for the binary case.

For later reference, we write the normalized symbol likelihoods as follows. Restricting ourselves (for the moment) to the real case, we have

$$\tilde{p}(A_m) \triangleq \frac{f(y|A_m)}{\sum_{\ell=0}^{M-1} f(y|A_\ell)} \tag{2}$$

$$= \frac{\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-A_m)^2}{2\sigma^2}}}{\sum_{\ell=0}^{M-1} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-A_\ell)^2}{2\sigma^2}}} \tag{3}$$

$$= \left(\sum_{\ell=0}^{M-1} e^{\frac{2y(A_\ell - A_m) + (A_m^2 - A_\ell^2)}{2\sigma^2}} \right)^{-1} \tag{4}$$

In Section II, we describe the new circuits for the computation of symbol likelihoods (2) for 1-dimensional (i.e., real) constellations. In Section III, we describe the extension to general (not necessarily square) 2-dimensional (i.e., complex) constellations. In Section IV, we show how bit-likelihoods are easily obtained for arbitrary bit-to-symbol mappings.

II. SYMBOL LIKELIHOODS FOR ONE-DIMENSIONAL CONSTELLATIONS

For the following circuits all the transistors are assumed to function as voltage controlled current sources with an exponential characteristic between the output current and the control voltage. This assumption holds both for bipolar transistors as well as for MOS transistors in weak inversion. We use the following notation for MOS transistors in weak inversion (operating in the active region):

$$I_{\text{D}} = I_{\text{D0}} e^{\frac{V_{\text{GS}}}{nU_{\text{T}}}}, \tag{5}$$

where I_D is the drain current, I_D0 is a technology-dependent constant, V_GS is the gate-source voltage, n the slope-factor of a MOS transistor and U_T the thermal voltage [8].

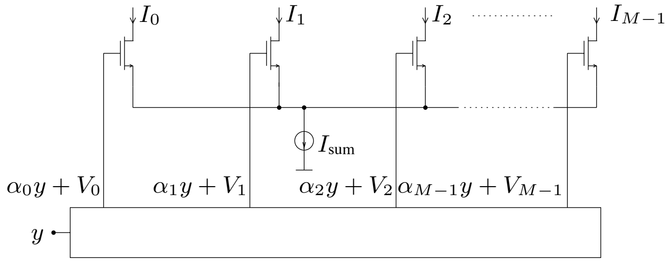

Fig. 2. Generic circuit for determining the soft symbols of a PAM signal.

As illustrated in Fig. 2, the input to the proposed circuit, y (e.g., an input voltage), is amplified with various gain-factors α_i and shifted by offsets V_i. The resulting voltages, $\alpha_i y + V_i$, are then applied to the gates of the transistors of this circuit.

The equations corresponding to the normalized currents $\frac{I_m}{I_\mathrm{sum}}$ of the circuit shown in Fig. 2 are given in (7).

$$\frac{I_m}{I_\mathrm{sum}} = \frac{I_\mathrm{D0} \cdot e^{\frac{\alpha_m V_\mathrm{in} + V_m}{nU_\mathrm{T}}}}{\sum_{j=0}^{M-1} I_\mathrm{D0} \cdot e^{\frac{\alpha_j V_\mathrm{in} + V_j}{nU_\mathrm{T}}}} \tag{6}$$

$$= \left(\sum_{j=0}^{M-1} e^{\frac{(\alpha_j - \alpha m) V_\mathrm{in} + (V_j - V_m)}{nU_\mathrm{T}}} \right)^{-1} \tag{7}$$

Note the similarity of (7) and (4). Indeed, by choosing α_i and V_i such that for all $0 \le i, j \le M-1$,

$$\alpha_j - \alpha_i = \frac{A_j - A_i}{\sigma^2} \cdot \frac{y}{V_\mathrm{in}} \cdot nU_\mathrm{T} \tag{8}$$

and

$$V_j - V_i = \frac{A_i^2 - A_j^2}{2\sigma^2} \cdot nU_\mathrm{T}, \tag{9}$$

we obtain

$$\frac{(\alpha_j - \alpha_i)V_\mathrm{in} + (V_j - V_i)}{nU_\mathrm{T}} \tag{10}$$

$$= \frac{2(A_j - A_i)y + (A_i^2 - A_j^2)}{2\sigma^2}, \tag{11}$$

which means that (7) and (4) coincide.

It must be emphasized that the absolute value of the voltages V_i, as well as the absolute value of the gain-factors α_i, are not fixed, only the gain-differences are determined by (8) and the voltage-differences by (9).

The gate voltages, $\alpha_i y + V_i$, can easily be obtained by using circuits containing operational amplifiers; some example circuits are described in [9].

It is obvious from (8) and (9) that the values of the parameters α_i and V_i depend on both the signal-to-noise ratio and on the temperature. These parameters should therefore be adjusted by some (digital or analog) control loop.

III. Symbol Likelihoods for Two-Dimensional Constellations

The circuits presented in the previous section can easily be extended to compute the symbol likelihoods for QAM and for more general N-dimensional constellations. In the case of QAM, the symbols X, Y and Z in (1), as well as the constellation symbols $s_{m,n}$ are complex. An example of a QAM constellation is shown in Fig. 3.

Let X_I and X_Q be the in-phase and the quadrature component (the real and the imaginary part) of X, respectively. Analogously, we write s_I and s_Q, and Y_I and Y_Q. The transmission of one QAM symbol X may be viewed as the transmission of the two PAM symbols X_I and X_Q. For any constellation point $s_{m,n}$, we have

$$f_{Y|X}(y|s_{m,n}) = f_{Y_\mathrm{I}|X_\mathrm{I}}(y_\mathrm{I}|s_\mathrm{I}) \cdot f_{Y_\mathrm{Q}|X_\mathrm{Q}}(y_\mathrm{Q}|s_\mathrm{Q}) \tag{12}$$

Each of the two factors $f_{Y_\mathrm{I}|X_\mathrm{I}}(y_\mathrm{I}|s_\mathrm{I})$ and $f_{Y_\mathrm{Q}|X_\mathrm{Q}}(y_\mathrm{Q}|s_\mathrm{Q})$ can be obtained by the circuit of Fig. 2. The product (12) may then be computed (simultaneously for all QAM constellation points $s_{m,n}$) with a multiplier matrix circuit [1].

If not all symbols of an M-QAM signal are transmitted, e.g., if a cross-shaped constellation is used, the same structure as for the underlying rectangular M-QAM constellation can be used. The currents related to the omitted symbols can be dissipated and the remaining outputs can be rescaled with a current-scaling circuit [10].

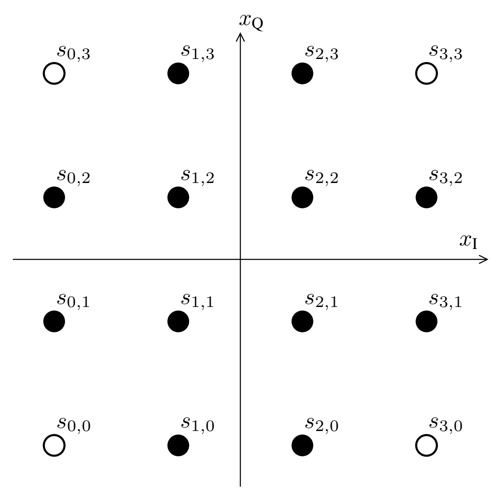

Fig. 3. Rectangular 16-QAM signal constellation and, with the empty dots neglected, 12-QAM cross-constellation.

As an example, the circuit for detecting the soft symbols of a 16-QAM signal (with the constellation given in Fig. 3) can be obtained by the lower part of the circuit shown in Fig. 4. As mentioned before, it consists of three parts: two circuits compute the soft symbols for a 4-PAM signal, extracted from the I-phase or the Q-phase of the QAM-signal, respectively. The matrix multiplier then computes all the symbol metrics out of the metrics from the two phases.

The soft symbols of a 12-QAM signal (with a cross-constellation, cf. Fig. 4) can be obtained by adding the top part ("16-QAM to 12-QAM Converter") of the circuit given in Fig. 4.

IV. From Symbol Likelihoods to Bit Likelihoods

In trellis coded modulation [11] and bit-interleaved coded modulation [12], [13], the likelihoods $f(Y = y|X = s_i)$ of several symbols s_i are combined to form subset likelihoods or bit likelihoods (also called "label metrics"), which are then used by the decoder. For example for a 4-PAM constellation, each symbol can be labeled by two binary numbers as it is shown in Figure 5.

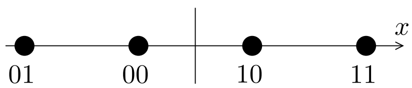

Fig. 5. Signal space diagram for symmetric 4-PAM signals labeled with two binary numbers.

The metrics of the least significant bit, μ_{lsb}, and the metric of the most significant bit, μ_{msb}, can be obtained by marginalizing the symbol likelihoods, i.e.:

$$\mu_{\text{lsb}} = [\mu_{\text{lsb}}(0), \mu_{\text{lsb}}(1)] \quad (13)$$
$$\mu_{\text{lsb}}(0) = f(Y = y|X = 00) + f(Y = y|X = 10) \quad (14)$$
$$\mu_{\text{lsb}}(1) = f(Y = y|X = 01) + f(Y = y|X = 11) \quad (15)$$

$$\mu_{\text{msb}} = [\mu_{\text{msb}}(0), \mu_{\text{msb}}(1)] \quad (16)$$
$$\mu_{\text{msb}}(0) = f(Y = y|X = 00) + f(Y = y|X = 01) \quad (17)$$
$$\mu_{\text{msb}}(1) = f(Y = y|X = 10) + f(Y = y|X = 11) \quad (18)$$

The circuit computing the currents that are proportional to the metrics μ_{lsb} and μ_{msb}, I_{lsb} and I_{msb}, respectively, is given in Figure 6. The additions of the likelihoods in (14), (15) and (17), (18) are realized by adding the currents that correspond to these likelihoods. It is clear that this scheme is not limited to PAM constellations or to two-bit binary numbers, but can be readily extended to more complicated scenarios.

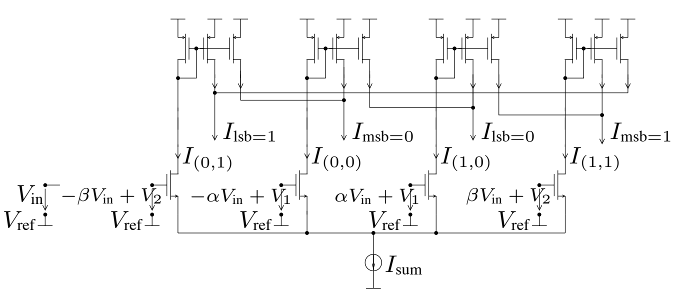

Fig. 6. Circuit used for detecting the soft-symbols of a 4-PAM signal and then computing the label metrics μ_{lsb} and μ_{msb}.

V. Conclusion

We have presented new analog circuits to convert the output of a matched filter into symbol- and bit-likelihoods suitable as input to analog decoders. The new circuits are suitable for many constellations including PAM and QAM and for arbitrary bit-to-symbol mappings.

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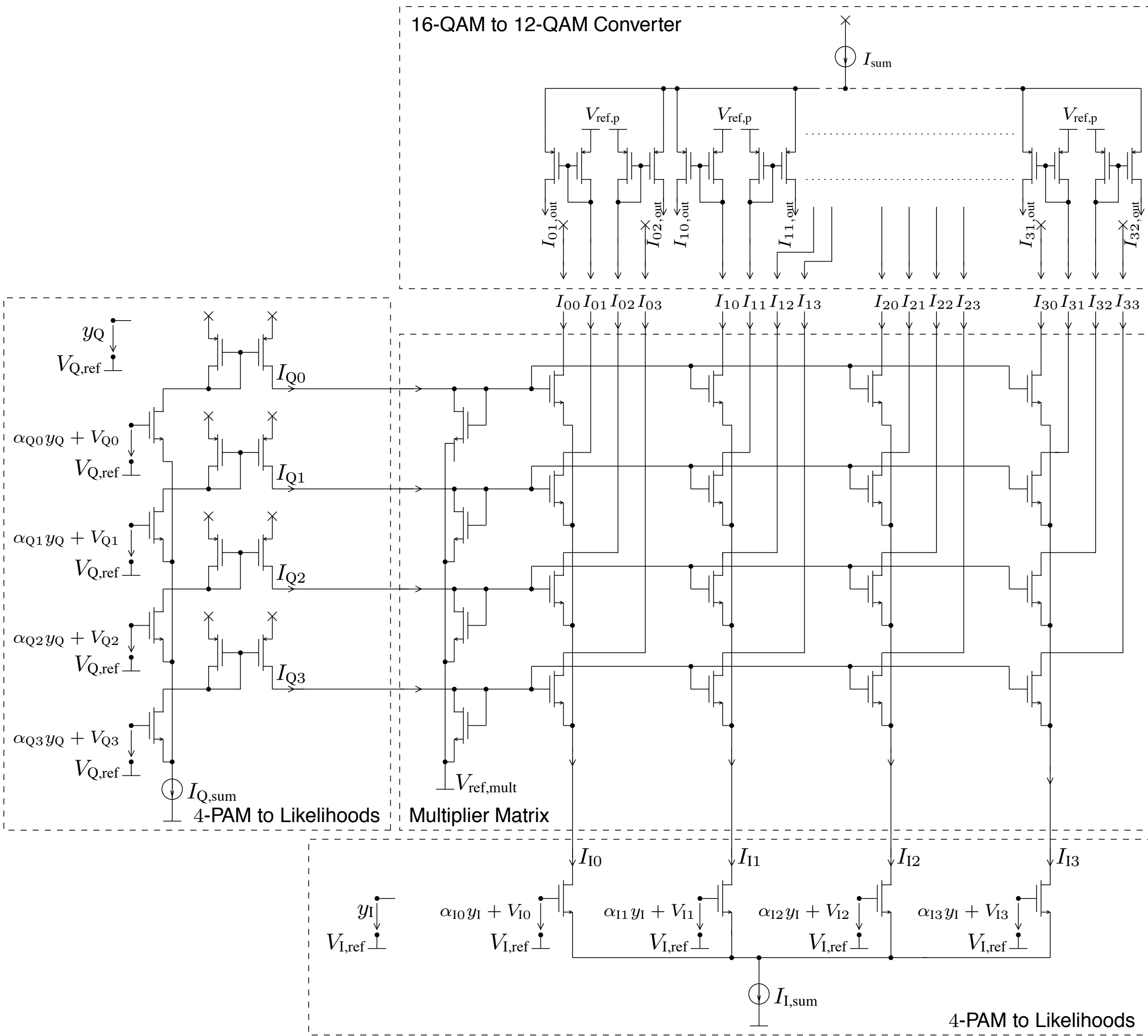

Fig. 4. Soft symbol detection circuit for rectangular 16-QAM signals.