

An Exact and Direct Analytical Method for the Design of Optimally Robust CNN Templates

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Abstract—In this paper, we present an analytical design approach for the class of bipolar cellular neural networks (CNN's) which yields optimally robust template parameters. We give a rigorous definition of absolute and relative robustness and show that all well-defined CNN tasks are characterized by a finite set of linear and homogeneous inequalities. This system of inequalities can be analytically solved for the most robust template by simple matrix algebra. For the relative robustness of a task, a theoretical upper bound exists and is easily derived, whereas the absolute robustness can be arbitrarily increased by template scaling. A series of examples demonstrates the simplicity and broad applicability of the proposed method.

Index Terms—Cellular neural networks (CNN's), robustness, template design.

I. INTRODUCTION

A. Cellular Neural Networks and Template Learning

IN THIS paper, we consider the class of single-layer spatially invariant cellular neural networks (CNN's) with a neighborhood radius of one, defined in [1]. The dynamics of the network is governed by a system of $n = MN$ differential equations

$$\begin{aligned} \frac{dx_{ij}(t)}{dt} = & -x_{ij}(t) + \sum_{k,l \in \mathcal{N}_{ij}} (a_{k-i,l-j} \text{sat}(x_{kl}(t)) \\ & + b_{k-i,l-j} u_{kl}) + I + \partial_{ij}, \end{aligned} \quad (1)$$

$(i, j) \in \{1, \dots, M\} \times \{1, \dots, N\}$

where $\mathcal{N}_{ij}$ denotes the neighborhood of the cell C_{ij} , a_{kl} and b_{kl} the feedback and the control template parameters, respectively. Since the cells on the margins do not have a complete set of regular neighbors, the CNN is assumed to be surrounded by a virtual ring of cells with constant input and output. Their contribution is subsumed under ∂_{ij} . $\text{sat}(\cdot)$ is the piecewise linear saturation function

$$\text{sat}(x) = \frac{1}{2}(|x+1| - |x-1|) \quad (2)$$

and $y_{ij}(t) = \text{sat}(x_{ij}(t))$ is referred to as the output of C_{ij} .

In this class of CNN's, the template set $\mathcal{T} = \{A, B, I\}$ is fully specified by 19 parameters; $A = \{a_{kl}\}$ and $B = \{b_{kl}\}$ are 3×3 matrices. We restrict ourselves to the class of bipolar

CNN's, where $\mathbf{u}, \mathbf{y}(t \rightarrow \infty) \in \mathbb{B}^n$ and $\mathbf{x}(0) \in \mathbb{B}_0^n$ for $\mathbb{B} = \{-1, 1\}$ and $\mathbb{B}_0 = \{-1, 0, 1\}$.

The problem of template design or template learning is a key topic in CNN research. The methods which have been investigated since the inception of the CNN may be classified as analytical methods [2]–[4], local learning algorithms [5]–[7], and global learning algorithms [8], [9]. The analytical approaches are based upon a set of local rules characterizing the dynamics of a cell, depending on its neighboring cells. These rules are transformed into an affine set of inequalities which must be solved to get correctly operating templates. Local learning algorithms are derived from training methods developed for other neural networks such as multilayer perceptrons, and their global counterparts mostly use stochastic optimization techniques such as genetic algorithms [8] or simulated annealing [9].

Analog very large scale integration (VLSI) implementations of the network equation (1) have a number of limitations that need to be taken into account in the theory of CNN's in order to guarantee correct and efficient operation of analog VLSI hardware. Template parameters can only be realized with a precision of typically 5–10% of the nominal values, and usually only a discrete set of possible values is available [10], [11]. The requirement that a template set fulfills a given task reliably under these circumstances poses additional obstacles to template design. The problem of synthesizing such robust templates has been successfully attacked using genetic algorithms [12], or by means of hybrid approaches where stochastic optimization techniques were combined with hill climbing algorithms [13]. However, they are computationally very expensive and yield little insight into the dynamics of the resulting templates. Analytical methodologies for the design of robust parameters deal with the problem of solving a system of affine inequalities for the best or, at least, a sufficiently robust template [4], [14]–[17]. The proposed algorithms include linear programming or relaxation methods which entail considerable effort, often without yielding guaranteed optimal solutions. In this paper, we show that after a small translation of the template space, the set of inequalities becomes homogeneous, and we propose the application of the matrix-vector notation to solve the CNN design problem, very generally, using simple matrix algebra. Furthermore, our approach provides informative insight into the interaction of the parameters.

Stability issues of CNN's [1], [18]–[22] are beyond the scope of this paper. If the system of inequalities is consis-

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tent, does not contain any loops,¹ and encompasses all cell configurations at the desired equilibria then its solution will be stable.

B. Locally Regular Template Sets

The template design method we propose is applicable to the important class of locally regular templates which is characterized in the following.

Definition 1: A CNN template set is locally regular if its operation can be characterized by a set of time-invariant local rules. The operation itself is then also called locally regular.

A local rule prescribes whether the state's derivative of a cell $\dot{x}_{ij}(t)$ is to be negative or positive for a particular bipolar configuration of the input and output values of the neighboring cells.

Definition 2: A cell C_{ij} is directly connected to a cell C_{mn} if $|i - m| \leq 1$ and $|j - n| \leq 1$, i.e., $m, n \in \mathcal{N}_{ij}$ and $a_{i-m, j-n} \neq 0$.

Definition 3: A linear cell is a cell for which $|x(t)| \leq 1$ or, equivalently, $x(t) = y(t)$ holds.

Lemma 4: A template is locally regular if and only if, for all linear cells, there is no other directly connected linear cell.

Proof: ($\Rightarrow$) Let C_L be a linear cell. For a locally regular template, all cells that are directly connected to C_L must have constant output. The dynamics of $x_L(t)$ in the linear region is then governed by $\dot{x}_L(t) = x_L(t)(a_c - 1) + q$ where q comprises the contributions from the neighbor's output values from the input, bias, and boundary which are all constant by assumption as long as C_L is linear. $a_c = a_{0,0}$ is the center element of the A -template. For bipolar tasks $a_c > 1$ is assumed. The solution then is a single exponential function with a positive argument, which guarantees that the equilibrium lies in the saturation region. Hence, the sign of $\dot{x}_L(t)$ is determined by the bipolar output values of the neighboring cells and cannot change while in the linear region, which implies that the template is locally regular.

($\Leftarrow$) If any cell C_L and a directly connected cell are linear at the same time, the dynamics $x_L(t)$ is described by a time-varying differential equation since it is influenced by its linear neighbor's nonbipolar and time-varying output. Thus the sign of $\dot{x}_L(t)$ cannot be determined by time-invariant local rules. ■

Remarks:

- The system of inequalities is valid during the whole transient, not only at $t = 0$. It does not matter whether a certain configuration of neighboring cells $\mathcal{C}$ occurs at the beginning of a transient or at a later time $t_0 > 0$. To see this, we consider two state trajectories $x_1(t)$ and $x_2(t)$. Positive initialization $x_1(0) = x_2(0) = 1$ and a neighborhood with a configuration $\mathcal{C}_A$ and $\mathcal{C}_B$, respectively, are assumed. The local rule for $\mathcal{C}_A$ may prescribe a negative derivative, $\dot{x}_1(0) = -1 + a_c + q_A < 0$, whereas $\dot{x}_2(0) = -1 + a_c + q_B > 0$. q_A and q_B subsume the contribution from the neighbors and the bias for the configurations $\mathcal{C}_A$ and $\mathcal{C}_B$, respectively. Clearly, $x_1(t)$

tends to an (intermediate) equilibrium $x_1^* = -a_c + q_A < -1$ and $x_2(t)$ to $x_2^* = a_c + q_B > 1$. Suppose the configuration around the second cell changes from $\mathcal{C}_B$ to $\mathcal{C}_A$ at $t = t_0$. The derivative $\dot{x}_2(t_0) = -x_2^* + a_c + q_A$ is negative and thus satisfies the local rule for $\mathcal{C}_A$. Hence, the fact that such intermediate equilibria lie in the saturation region and the negative sign of $x(t)$ on the right side of the CNN, (1) guarantees the validity of the local rules during the whole transient. It is not important how precisely these equilibria x^* are reached before the configuration changes and, accordingly, a new local rule applies.

- All uncoupled templates, where only the center element of the A template is nonzero, are inherently locally regular since there is no influence from the neighbor's output. Most of the well-known propagation-type tasks such as shadowing, connected component detection, hole filling, and global connectivity detection are locally regular as well. A counterexample is the Laplacian template [23], which is locally irregular since its behavior cannot be described by a set of local rules.
- Local regularity does not imply any symmetry or sign symmetry of the A template.

II. THE ROBUSTNESS OF A TEMPLATE SET

A. Absolute and Relative Robustness

The robustness of a CNN template set is a measure which quantifies the degree by which a template set can be altered while still producing the desired output. In programs for CNN VLSI chips it is crucial that all templates have a certain degree of robustness since their values cannot be guaranteed to be reproduced exactly by the analog circuit.

Various definitions of robustness [12], [16], [17], [24] exist. We define the vector $\mathbf{p}$ to contain all m nonzero² entries in a template set $\mathcal{T}$ with the center element of the A template as its first element ($p_1 := a_c$) and the other $m - 1$ elements in arbitrary order. We refer to the final output of a CNN programmed with $\mathbf{p}$ by $\mathbf{y}_\infty(\mathbf{p})$.

Definition 5: The absolute robustness ϵ of a template set is

$$\epsilon(\mathbf{p}) = \max_{\alpha} \{ \alpha \mid \mathbf{y}_\infty(\mathbf{p}) = \mathbf{y}_\infty(\mathbf{p} + \alpha \mathbf{1}^\pm) \quad \forall \mathbf{1}^\pm \in \mathbb{B}^m \}.$$

Hardware tolerance effects due to physical and manufacturing imperfections give rise to parameter errors roughly proportional to the absolute value of the respective parameter [25]. We therefore consider a relative robustness criterion.

Definition 6: The relative robustness D of a template set is

$$D(\mathbf{p}) = \max_{\alpha} \{ \alpha \mid \mathbf{y}_\infty(\mathbf{p} \circ (\mathbf{1} + \alpha \mathbf{1}^\pm)) = \mathbf{y}_\infty(\mathbf{p}) \quad \forall \mathbf{1}^\pm \in \mathbb{B}^m \}.$$

$\circ$ denotes componentwise vector multiplication.

¹A loop is when, e.g., some configuration of cells $\mathcal{C}_1$ is supposed to lead to a configuration $\mathcal{C}_2$ and vice versa.

²A zero template entry is assumed to be realized by omitting some circuitry, or by switching or disabling some controlled source, not by nulling. Zero template entries are therefore precise.

For the sake of clarity and mathematical tractability we define a slightly modified template vector $\tilde{\mathbf{p}}$ to be

$$\tilde{p}_1 := p_1 - 1 = a_c - 1, \quad \tilde{p}_i := p_i, \quad 2 \leq i \leq m$$

or, alternatively, $\tilde{\mathbf{p}} := \mathbf{p} - \mathbf{e}_1$ where $\mathbf{e}_1$ is the unit vector in direction of increasing a_c .

Locally regular CNN tasks are fully characterized by a set of $\hat{m}$ inequalities for $\dot{\mathbf{x}}(t)$. Utilizing the modified template vector $\tilde{\mathbf{p}}$ and matrix-vector notation, the region $\mathcal{R} \subset \mathbb{R}^m$ where a template set operates correctly is then defined to be

$$\mathcal{R} = \{\tilde{\mathbf{p}} \in \mathbb{R}^m \mid (\mathbf{K} \cdot \tilde{\mathbf{p}})_i > 0, \quad \forall 1 \leq i \leq \hat{m}\} \quad (3)$$

for a coefficient matrix $\mathbf{K} \in \mathbb{B}_0^{\hat{m} \times m}$ representing the different constellations of $\mathbf{u}$ and $\mathbf{x}(t)$. (If a negative derivative is prescribed, the sign of the corresponding row in $\mathbf{K}$ is adapted.) The strict greater than inequality may also be a greater than or equal to inequality for, at most, $\hat{m} - 1$ of these inequalities.³ Note that a value of zero in $\mathbf{K}$ is only possible in case of zero initialization or zero boundary values.

By means of the set

$$\mathcal{R}' = \{\tilde{\mathbf{p}} \in \mathbb{R}^m \mid (\mathbf{K} \cdot \tilde{\mathbf{p}})_i \geq 0, \quad \forall 1 \leq i \leq \hat{m}\} \supset \mathcal{R} \quad (4)$$

which includes the boundary of $\mathcal{R}$ we introduce the term safety margin. We will denote it by $\gamma(\mathcal{T})$ and formally define it to be

$$\gamma(\mathcal{T}) = \gamma(\tilde{\mathbf{p}}) = \min_{1 \leq i \leq \hat{m}} \{(\mathbf{K} \cdot \tilde{\mathbf{p}})_i\}, \quad \tilde{\mathbf{p}} \in \mathcal{R}'. \quad (5)$$

The absolute and the relative robustness may now be expressed as

$$\epsilon(\mathcal{T}) = \frac{\gamma(\mathcal{T})}{m} \quad \text{and} \quad D(\mathcal{T}) = \frac{\gamma(\mathcal{T})}{\|\mathcal{T}\|_1} \quad (6)$$

respectively.

It may seem tedious to establish a system (3) for a task with a high connectivity. To reduce the dimension m , the system can be recast in a form where parameters known *a priori* to be identical are represented by a single variable. Since most highly connected tasks exhibit such isotropic behavior, this results in a manageable system. The new matrix $\tilde{\mathbf{K}}$ will then be in $\mathbb{Z}^{\hat{m} \times \hat{m}}$.

B. Template Scaling

From (5) and (6) it follows that $\epsilon(q\tilde{\mathbf{p}}) = q\epsilon(\tilde{\mathbf{p}})$, i.e., by scaling the template vector $\tilde{\mathbf{p}}$ by a factor of q , we achieve proportionally higher absolute robustness (cf. [26]).

For the relative robustness we obtain

$$D(\tilde{\mathbf{p}}) = \frac{\gamma(\tilde{\mathbf{p}})}{\|\tilde{\mathbf{p}}\|_1 + 1} \Rightarrow D(q\tilde{\mathbf{p}}) = \frac{q\gamma(\tilde{\mathbf{p}})}{q\|\tilde{\mathbf{p}}\|_1 + 1} = \frac{\gamma(\tilde{\mathbf{p}})}{\|\tilde{\mathbf{p}}\|_1 + \frac{1}{q}}$$

where we have made use of the fact that $\|\mathcal{T}\|_1 = \|\tilde{\mathbf{p}}\|_1 + 1$. Hence, the relative robustness is strictly monotonically increasing with increasing q but it is upperbounded by $\gamma(\tilde{\mathbf{p}})/\|\tilde{\mathbf{p}}\|_1$.

³If none of the inequalities were strict, a template for which $\dot{\mathbf{x}} \equiv 0$ would be allowed. No operation would be performed, since the initial state would be the equilibrium.

III. DESIGN OF OPTIMALLY ROBUST TEMPLATES

With the results from the previous section it is now easily seen that template optimization with respect to relative robustness implies increasing $\gamma(\tilde{\mathbf{p}})$ while keeping $\|\tilde{\mathbf{p}}\|_1$ small. Template scaling by large factors does not improve the robustness significantly and has the disadvantage of resulting in larger template values, which may not be realizable on the CNN chip.

The design of a template with maximum robustness is in fact a design centering problem since $\tilde{\mathbf{p}}_{\text{opt}}$ is, in some sense, centered in $\mathcal{R}$. Formulated more precisely, the problem is to find a template set $\mathcal{T}_{\text{opt}}$ (or $\tilde{\mathbf{p}}_{\text{opt}}$) having the same safety margin in all its inequalities

$$(\mathbf{K} \cdot \tilde{\mathbf{p}}_{\text{opt}}(\gamma))_i = \gamma, \quad 1 \leq i \leq \hat{m} \quad (7)$$

assuming that the system of inequalities is nonredundant in the sense of the following definition.

Definition 7—Nonredundant Set of Inequalities: A system $(\mathbf{K}\mathbf{x})_i > 0$ is nonredundant if every row in $\mathbf{K}$ contributes to a diminution of the solution space or, equivalently, if no inequality in the system can be removed without affecting the solution space.

Lemma 14 in the Appendix shows how redundant inequalities are found.

Using the concepts introduced here, it emerges that the optimally robust template set can be calculated analytically in a rather elegant manner. The method of solving (7) depends on m , $\hat{m}$ and the rank of $\mathbf{K}$. We refer to a set of inequalities for which $m = \hat{m} = \text{rank } \mathbf{K}$ as a basic set (or system). Clearly, $\mathbf{K}$ is invertible in case of such basic sets. For the sake of simplicity and clarity, we restrict ourselves to this important class in the following theorem and treat all nonbasic systems in the Appendix.

Theorem 8—Optimally Robust Templates for Basic Systems: Assuming that

$$(\mathbf{K} \cdot \tilde{\mathbf{p}})_i > 0, \quad 1 \leq i \leq m, \quad \mathbf{K} \in \mathbb{B}_0^{m \times m}, \quad \tilde{\mathbf{p}} \in \mathbb{R}^m \quad (8)$$

is a set of nonredundant inequalities characterizing a CNN task, the optimally robust template vector $\tilde{\mathbf{p}}_{\text{opt}}$ as a function of a scaling parameter q is

$$\tilde{\mathbf{p}}_{\text{opt}}(q) = q\mathbf{K}^{-1}\mathbf{1}^m. \quad (9)$$

($\mathbf{1}^m$ denotes the vector in $\mathbb{R}^m$ with all its components +1.) Its relative robustness is

$$D(\tilde{\mathbf{p}}_{\text{opt}}(q)) = \frac{1}{\|\mathbf{K}^{-1}\mathbf{1}^m\|_1 + \frac{1}{q}} \quad (10)$$

where q equals the safety margin and the theoretical maximum $\hat{D}$ for the achievable robustness is

$$\hat{D} = \lim_{q \rightarrow \infty} D(\tilde{\mathbf{p}}_{\text{opt}}(q)) = \frac{1}{\|\mathbf{K}^{-1}\mathbf{1}^m\|_1}. \quad (11)$$

An optimally robust template set is a template set that has maximum robustness for a given norm $\|\mathcal{T}\|_1$.

Proof: It follows directly from (7) that the optimum template set is the product of the inverted coefficient matrix and the vector of identical safety margins for all inequalities. The robustness is derived from (6). ■

Definition 9—The Principal Axis of a CNN Task: Assuming that a CNN task is characterized by a set of inequalities (3), the principal axis points in the direction of the unit vector $\tilde{\mathbf{p}}_{\text{opt}}(1)/\|\tilde{\mathbf{p}}_{\text{opt}}(1)\|_1$ and contains all templates $\tilde{\mathbf{p}} = \tilde{\mathbf{p}}_{\text{opt}}(q)$ with $q > 0$.

The principal axis of a task comprises all templates that are optimal with respect to robustness. Hence, scaling such a template does not affect its property of being optimal.

Corollary 10—Unboundedness of $\mathcal{R}$: $\mathcal{R}$ is infinite. For basic systems, it is spanned by

$$\mathbf{K}^{-1}\boldsymbol{\lambda}, \quad \forall \boldsymbol{\lambda} \in (\mathbb{R}^+)^m. \quad (12)$$

Proof: In every case $\mathcal{R}$ is not bounded along the principal axis. For basic systems (8) is satisfied for any vector $\boldsymbol{\lambda}$ with solely positive components. ■

For all CNN chips, there is an upper bound for $\|\mathcal{T}\|_1$. The next corollary shows how to find the optimum template under this constraint.

Corollary 11—Optimum Template for $\|\mathcal{T}\|_1 \leq \beta$: Under the constraint $\|\mathcal{T}\|_1 \leq \beta$ the optimum template is

$$\tilde{\mathbf{p}}_{\text{opt}}|_{\|\mathcal{T}\|_1 \leq \beta} = (\beta - 1) \frac{\tilde{\mathbf{p}}_{\text{opt}}(1)}{\|\tilde{\mathbf{p}}_{\text{opt}}(1)\|_1} \quad (13)$$

with a relative robustness of

$$D(\tilde{\mathbf{p}}_{\text{opt}}|_{\|\mathcal{T}\|_1 \leq \beta}) = \frac{1}{\|\tilde{\mathbf{p}}_{\text{opt}}(1)\|_1} \left(1 - \frac{1}{\beta}\right). \quad (14)$$

Proof: Expressed by $\tilde{\mathbf{p}}$ the constraint is $\gamma\|\tilde{\mathbf{p}}_{\text{opt}}(1)\|_1 + 1 \leq \beta$. Solving for γ and inserting (6) and (9) yields the above results. ■

Given the constraint β , (14) tells us directly how far we are from the theoretical optimum. For $\beta = 10$, for example, we achieve 90% of this upper bound.

Corollary 12—Initialization for Uncoupled Tasks: If, for an uncoupled bipolar CNN task, the initial state is chosen to be $\mathbf{x}(0) = \pm \mathbf{1}$, $\mathbf{x}(0) = \mathbf{0}$, or $\mathbf{x}(0) = \mathbf{u}$ then the space of optimally robust templates according to (7) is not necessarily a single point, but may have dimension ≥ 1 . Depending on the initialization, there is a degree of freedom in the $(\tilde{p}_1, b_c)$ or in the $(\tilde{p}_1, I)$ plane, subject to the constraint $\tilde{p}_1 \geq 0$ in order to guarantee bipolar output ($\tilde{p}_1 := a_c - 1$).

$\mathbf{x}(0) = \mathbf{0}$: $\tilde{p}_1 = 0$ is the optimum choice.

Proof: $\tilde{p}_1$ does not appear in the parameter vector $\tilde{\mathbf{p}}$. Any $\tilde{p}_1 \geq 0$ does not influence the functionality of the template. Thus, we set $\tilde{p}_1 = 0$ for maximum relative robustness.

$\mathbf{x}(0) = -\mathbf{1}$. Let $c := -\tilde{p}_1 + I$. From (7) we get only a value for c , but not for the individual parameters $\tilde{p}_1$ and I . If $c < 0$ we may choose $\tilde{p}_1 \in [0, -c]$ and $I = \tilde{p}_1 + c$ without affecting the robustness. If $c \geq 0$ then $\tilde{p}_1 = 0$ and $I = c$ is the only optimum solution.

Proof: In the matrix $\mathbf{K}$ $-\tilde{p}_1$ and I have the same coefficient in every row, since for initialization with -1 , $(-1)(-\tilde{p}_1)$ plays the role of an additional bias at $t = 0$. Hence, the matrix $\mathbf{K}$ is singular and $\tilde{p}_1$ and I are only determined by the value of c and $\tilde{p}_1 > 0$. For maximum relative robustness, $\tilde{p}_1 + |I| = \tilde{p}_1 + |\tilde{p}_1 + c|$ has to be minimal, which results in the above solution space.

$\mathbf{x}(0) = \mathbf{1}$. Let $c := \tilde{p}_1 + I$. As in the previous case, only c is determined by (7). If $c > 0$ all $\tilde{p}_1 \in [0, c]$ $I = c - \tilde{p}_1$ are optimum values, whereas for $c \leq 0$ only one optimum solution exists, that is, $\tilde{p}_1 = 0$, $I = c$.

Proof: For positive initialization $\tilde{p}_1$ can be considered as additional bias at $t = 0$, and $\tilde{p}_1$ and I have the same coefficients in $\mathbf{K}$. Thus, only $c = \tilde{p}_1 + I$ is fixed by (7). Depending on c the minimization of $|\tilde{p}_1| + |I|$ yields the above solution(s).

$\mathbf{x}(0) = \mathbf{u}$. Let $c := \tilde{p}_1 + b_c$. The optimum solutions for $\tilde{p}_1$ and b_c are $\tilde{p}_1 \in [0, c]$ $b_c = c - \tilde{p}_1$ if $c > 0$ and $\tilde{p}_1 = 0$, $b_c = c$ if $c < 0$.

Proof: Similar reasoning as in the previous case applies. Simply replace I by b_c since $\tilde{p}_1$ and b_c have the same coefficients in $\mathbf{K}$ for input initialization. ■

Remark: When applying Theorem 8 to uncoupled tasks, it is advantageous not to include $\tilde{p}_1$ (and b_c or I , respectively) in the matrix $\mathbf{K}$, but just c (as defined in Corollary 12) in order to get a regular matrix and then to determine $\tilde{p}_1$ and b_c or I , respectively, after having calculated the optimum value for c .

However, by using *QR* decomposition, Theorem 15 in the Appendix would of course lead to the same result with a parameter vector including $\tilde{p}_1$, b_c , and I .

IV. EXAMPLES

Intentionally, we make only minimal use of *a priori* knowledge, and merely assume symmetry of isotropic tasks. For those examples, we use the reduced coefficient matrix $\tilde{\mathbf{K}} \in \mathbb{Z}^{\hat{m} \times m}$ instead of $\mathbf{K} \in \mathbb{B}_0^{\hat{m} \times m}$. We consider the actual image to be in black (+1) on a white (−1) background. The boundary condition for both state and input is assumed to be −1 throughout this section.

The functionality of the templates proposed in this section may be verified using the simulator available on the World Wide Web⁴ [27].

Example 1—Uncoupled Horizontal Line Detection:

Template Prototype:

$$A = [\tilde{p}_1 + 1], \quad B = [s \quad b_c \quad s], \quad I = z, \quad \mathbf{x}(0) = \mathbf{u}.$$

Definition of the Task: With $c := \tilde{p}_1 + b_c$, we get

	$u = y(0) \rightarrow y_\infty$	$\dot{x}(0)$
(1)	○ ● ○ → ○ ○ ○	$-c + 2s - z > 0$
(2)	○ ● ● → ○ ● ●	$c + z \geq 0$
(3)	● ○ ● → ● ○ ●	$c - 2s - z \geq 0$
(4)	● ○ ○ → ● ○ ○	$c - z \geq 0$

⁴http://www.isi.ee.ethz.ch/~haenggi/CNNsim_adv.html

$$\begin{bmatrix} -1 & 2 & -1 \\ 1 & 0 & 1 \\ 1 & -2 & -1 \\ 1 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} c \\ s \\ z \end{bmatrix} = \gamma \mathbf{1}^4.$$

Removal of Redundant Inequalities: The last row vector $\mathbf{k}_4$ is the sum of the other three and should be eliminated. No additional nonredundant inequalities (i.e., cell configurations) can be included.

Solution:

$$\tilde{\mathbf{p}}_{\text{opt}} = \gamma \tilde{\mathbf{K}}^{-1} \mathbf{1}^3 \implies c = 2\gamma, \quad s = \gamma, \quad z = -\gamma$$

$\tilde{p}_1 \in [0, 2\gamma]$ leads to the extrema

$$A = [2\gamma + 1], \quad B = [\gamma \ 0 \ \gamma], \quad I = -\gamma$$

and

$$A = [1], \quad B = [\gamma \ 2\gamma \ \gamma], \quad I = -\gamma.$$

The maximum achievable robustness is $1/5 = 20\%$.

Example 2—Shadowing:

Template Prototype:

$$A = [s \ \tilde{p}_1 + 1 \ q], \quad B = 0, \quad I = z; \quad \mathbf{x}(0) = \mathbf{u}.$$

Definition of the Task:

	$y(t) \rightarrow y(t+T)$	$\dot{x}(t)$
(1)	○○● → ○●●	$-s - \tilde{p}_1 + q + z > 0$
(2)	●○○ → ●●●	$s - \tilde{p}_1 + q + z > 0$
(3)	●●● → ●●●	$s + \tilde{p}_1 + q + z \geq 0$
(4)	○●● → ○●●	$-s + \tilde{p}_1 + q + z \geq 0$
(5)	○○○ → ○○○	$s + \tilde{p}_1 + q - z \geq 0$
(6)	●○○ → ●○○	$-s + \tilde{p}_1 + q - z \geq 0$
(7)	●●○ → ●●○	$s + \tilde{p}_1 - q + z \geq 0$
(8)	○●○ → ○●○	$-s + \tilde{p}_1 - q + z \geq 0$

$$\begin{bmatrix} -1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \tilde{p}_1 \\ s \\ q \\ z \end{bmatrix} = \gamma \mathbf{1}^8.$$

Removal of Redundant Inequalities: We find $\mathbf{k}_3 = \mathbf{k}_2 + \mathbf{k}_6 + \mathbf{k}_7$ and $\mathbf{k}_4 = \mathbf{k}_2 + \mathbf{k}_6 + \mathbf{k}_8$. The remaining six inequalities are nonredundant.

Solution: Since we end up with a system with $\hat{m} = 6 > m = 4$ this is not a basic set and we have to solve it in the least squares sense (cf. Case III in Theorem 15) which yields

$$\tilde{\mathbf{p}}_{\text{opt}} = (\mathbf{K}^t \mathbf{K})^{-1} \mathbf{K}^t \mathbf{1}^6 \implies \tilde{p}_1 = q = z = \gamma, \quad s = 0$$

which is, in fact, the well-known shadowing template. The maximum achievable robustness is $1/3 = 33\%$.

Remark: This method may also be applied to check whether there exists a shadowing template with a symmetrical A template and, if so, to determine the most robust solution. With the prototype

$$A = [s \ \tilde{p}_1 + 1 \ s], \quad B = [q \ b_c \ r], \quad I = z, \quad \mathbf{x}(0) = -\mathbf{1}$$

and $c := \tilde{p}_1 - z$ we find the nonredundant set

u	$y(t) \rightarrow y(t+T)$	$\dot{x}(t)$
●●●	○○○ → ○●○	$-c - 2s + q + b_c + r > 0$
●●○	○○○ → ○●○	$-c - 2s + q + b_c - r > 0$
●○○	●○○ → ●○○	$c - q + b_c + r \geq 0$
○○○	○○○ → ○●●	$-c - q - b_c - r > 0$
●○○	●○○ → ●●●	$-c + 2s + q - b_c - r > 0$

Evaluating $\tilde{\mathbf{p}}_{\text{opt}} = \gamma \tilde{\mathbf{K}}^{-1} \mathbf{1}^5$ and minimizing $|\tilde{p}_1| + |I|$ yields

$$A = [\gamma \ 1 \ \gamma], \quad B = [-\gamma \ 2\gamma \ 0], \quad I = 2\gamma$$

with a maximum robustness of $1/7 \approx 14.3\%$.

Example 3—Connected Component Detection:

Template Prototype:

$$A = [s \ \tilde{p}_1 + 1 \ q], \quad B = 0, \quad I = z, \quad \mathbf{x}(0) = \mathbf{u}.$$

Definition of the Task:

	$y(t) \rightarrow y(t+T)$	$\dot{x}(t)$
(1)	●○○ → ●●○	$-\tilde{p}_1 + s - q + z > 0$
(2)	○○● → ○○●	$-\tilde{p}_1 + s - q - z > 0$
(3)	●○○ → ●○○	$\tilde{p}_1 - s - q - z \geq 0$
(4)	○○○ → ○○○	$\tilde{p}_1 - s - q + z \geq 0$
(5)	●●● → ●●●	$\tilde{p}_1 + s + q + z \geq 0$
(6)	○○○ → ○○○	$\tilde{p}_1 + s + q - z \geq 0$
(7)	●●○ → ●●○	$\tilde{p}_1 + s - q + z \geq 0$
(8)	○○○ → ○○○	$\tilde{p}_1 + s - q - z \geq 0$

Removal of Redundant Inequalities: We eliminate the last two rows since $\mathbf{k}_7 = \mathbf{k}_1 + \mathbf{k}_4 + \mathbf{k}_6$ and $\mathbf{k}_8 = \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_5$.

Solution: Solving this nonbasic system in the least squares sense (cf. Case III in Theorem 15) yields

$$\tilde{\mathbf{p}}_{\text{opt}} = (\mathbf{K}^t \mathbf{K})^{-1} \mathbf{K}^t \mathbf{1}^6 \implies \tilde{p}_1 = s = \gamma, \quad q = -\gamma, \quad z = 0,$$

with a maximum achievable robustness of $1/3 = 33\%$.

Example 4—Global Connectivity Detection: The global connectivity detector deletes any connected objects that are marked in the binary image. An object is marked by changing at least one pixel from black to white in the initial state. The output contains the unmarked objects only.

Template Prototype: This task exhibits isotropic behavior. The off-center entries in A and B are therefore *a priori* assumed to be identical. To ensure that the template will operate on both white images on a black background and black images on a white background, the bias is set to zero

$$A = \begin{bmatrix} 0 & s & 0 \\ s & \tilde{p}_1 + 1 & s \\ 0 & s & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & q & 0 \\ q & b_c & q \\ 0 & q & 0 \end{bmatrix},$$

$I = 0, \quad \mathbf{x}(0) = \text{bipolar image.}$

Definition of the Task:

	$u \quad y(t) \rightarrow y(t+T)$	$\dot{x}(t)$
(1)	  $\rightarrow$ 	$\tilde{p}_1 + b_c + 4s + 4q \geq 0$
(2)	  $\rightarrow$ 	$\tilde{p}_1 + b_c - 4s - 4q \geq 0$
(3)	  $\rightarrow$ 	$-\tilde{p}_1 - b_c - 2s - 4q > 0$
(4)	  $\rightarrow$ 	$\tilde{p}_1 - b_c - 4s - 4q \geq 0$
(5)	  $\rightarrow$ 	$\tilde{p}_1 + b_c + 2s + 2q \geq 0$
(6)	  $\rightarrow$ 	$-\tilde{p}_1 - b_c - 4q > 0$

Removal of Redundant Inequalities: Inequalities (1)–(4) constitute a regular nonredundant matrix $\tilde{\mathbf{K}}$. Other constellations such as $\mathbf{k}_5 = \frac{3}{4}\mathbf{k}_1 + \frac{1}{4}\mathbf{k}_2$ and $\mathbf{k}_6 = \mathbf{k}_1 + 2\mathbf{k}_3$ are clearly redundant.

Solution:

$$\tilde{\mathbf{p}}_{\text{opt}} = \gamma \tilde{\mathbf{K}}^{-1} \mathbf{1}^4 \implies \tilde{p}_1 = s = \gamma, \quad q = -\gamma, \quad b_c = 0$$

in agreement with the template found by stochastic optimization [13] where $\gamma = 2$ with a robustness of $2/19 = 10.5\%$. The maximum robustness for this task is $\gamma/\|\tilde{\mathbf{p}}_{\text{opt}}\|_1 = 1/9 = 11.1\%$.

V. SUMMARY AND CONCLUSIONS

In this paper, we have proposed an exact and analytical approach for the design of robust templates for the class of locally regular CNN tasks. Absolute and relative robustness is defined in a deterministic and easily reproducible manner. The desired task is characterized by a set of inequalities defining the subspace within which all correctly operating templates lie. We have shown that this system of inequalities can be solved directly for the optimally robust template; there is no need for an iterative algorithm. Furthermore, this analytical method provides insight into the dynamics of the CNN and the interaction between different template parameters.

Matrix-vector notation for the coefficient matrix and the vector of nonzero template entries is applicable. With a translation by -1 in the direction of the a_c axis the system turns out to be homogeneous. Redundant rows (i.e., equations that

are unnecessary since they do not impose any additional restrictions on the solution space) of the coefficient matrix are to be eliminated in a first step; these row vectors have the property of being positive linear combinations of others. Since the method includes the removal of these redundant inequalities it is not important to specify exactly the right inequalities and the right number thereof. One may include inequalities for all possible constellations or restrict oneself to a basic set. However, a small set is advantageous due to the smaller dimension of $\mathbf{K}$ and is, in fact, very often sufficient to fully characterize a task.

The safety margin specifies by what amount an inequality is satisfied. Optimally robust templates $\tilde{\mathbf{p}}_{\text{opt}}$ are those with the same safety margin γ in all nonredundant inequalities: $\mathbf{K}\tilde{\mathbf{p}}_{\text{opt}} = \gamma\mathbf{1}$. The solution depends on the dimension and the rank of $\mathbf{K}$. Often $\mathbf{K}$ is regular and we obtain simply $\tilde{\mathbf{p}}_{\text{opt}} = \gamma\mathbf{K}^{-1}\mathbf{1}$. If the system is overdetermined, we solve it in a least squares sense. If it is underdetermined, we may apply a QR decomposition or reduce the number of parameters. For every $\gamma > 0$ the solution is optimal in the sense that no other more robust template with a smaller or equal L_1 norm $\|\tilde{\mathbf{p}}\|_1$ exists.

With increasing γ the absolute and relative robustness increase strictly monotonically. The relative robustness is upperbounded for $\gamma \rightarrow \infty$. This upper bound is a property of the underlying task, the initial state, and the boundary value. It can be very easily determined and permits the optimization of the initial and boundary conditions for robustness.

In general, scaling a correctly operating template $\tilde{\mathbf{p}}$ by a positive factor always yields another valid solution, with the absolute robustness being scaled by the same factor. This implies that the subspace of the solutions for a given task is not bounded.

For VLSI implementations of the CNN's $\|T\|_1 = \|\tilde{\mathbf{p}}\|_1 + 1$ is constrained by some upper bound β . The proposed method directly yields the optimum solution for $\beta \leq \|T\|_1$ and specifies its degree of robustness.

APPENDIX

Definition 13—Positive Linear Combination: A positive linear combination of a set of vectors $\mathbf{x}_1, \dots, \mathbf{x}_m$ is a linear combination with solely nonnegative coefficients. To denote the subspace spanned by positive linear combinations of $\mathbf{x}_1, \dots, \mathbf{x}_m$ we use

$$\langle \mathbf{x}_1, \dots, \mathbf{x}_m \rangle^+.$$

Lemma 14—Determination of Redundant Inequalities: In a system $(\mathbf{K}\mathbf{k})_i > 0$, $\mathbf{K} \in \mathbb{R}^{\hat{m} \times m}$ the redundant inequalities are those which can be expressed as positive linear combinations of others. Hence, all row vectors $\mathbf{k}_i^t$ are redundant for which

$$\exists \lambda \in (\mathbb{R}_0^+)^{\hat{m}} \quad \text{such that} \quad \mathbf{k}_i^t = \sum_{\substack{j=1 \\ j \neq i}}^{\hat{m}} \lambda_j \mathbf{k}_j^t.$$

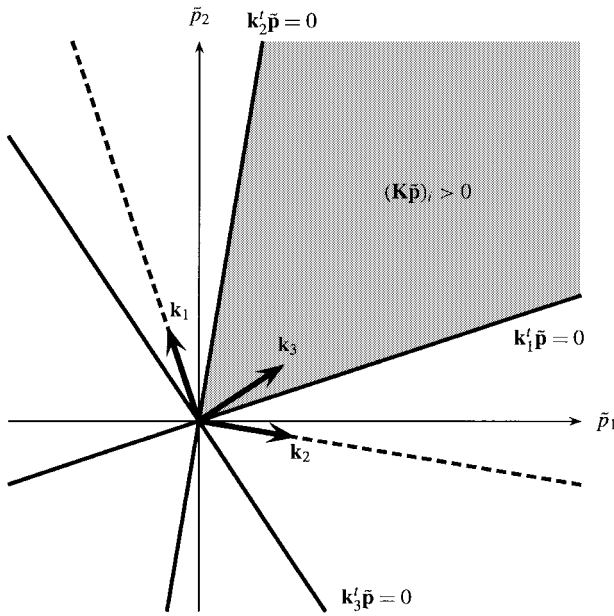


Fig. 1. Geometrical interpretation of Lemma 14. $\mathbf{k}_3$ is redundant.

Proof: Using $\mathbf{k}_1^t, \mathbf{k}_2^t, \dots, \mathbf{k}_m^t$ to denote the rows of $\mathbf{K}$ we note that the system is homogeneous in the sense that all hyperplanes $\mathbf{k}_i^t \mathbf{x}_i = 0$ intersect at the origin. If one of the inequalities, for example, $\mathbf{k}_i^t \tilde{\mathbf{p}} > 0$ can be expressed as the sum of positive multiples of some others, then $\mathbf{k}_i^t \tilde{\mathbf{p}} > 0$ is trivially satisfied and therefore redundant.

This argument has a geometrical interpretation. The vectors $\mathbf{k}_i$ normal to the hyperplanes $\mathbf{k}_i^t \tilde{\mathbf{p}} = 0$ point in the direction of the solution subspace. If one of them, say $\mathbf{k}_i$, points into the positive subspace spanned by the others $\{\mathbf{k}_1, \dots, \mathbf{k}_{i-1}, \mathbf{k}_{i+1}, \dots, \mathbf{k}_m\}^+$ this signifies that the inequality defined by $\mathbf{k}_i$ is redundant since it is always satisfied when the others hold. Hence, it can be eliminated without affecting the solution space. A possible constellation in $\mathbb{R}^2$ is depicted in Fig. 1, where $\mathbf{k}_3$ is redundant. ■

Remark: Matrix algebra may be used to determine which row vectors are positive linear combinations of others. In practice, however, the redundant equations can often be eliminated simply by inspection or excluded *a priori*.

Theorem 15—Optimally Robust Templates: Depending on m , $\hat{m}$, and the rank of $\mathbf{K}$ we consider different cases of the nonredundant system

$$(\mathbf{K} \cdot \tilde{\mathbf{p}})_i > 0, \quad 1 \leq i \leq \hat{m}, \quad \mathbf{K} \in \mathbb{B}_0^{\hat{m} \times m}, \quad \tilde{\mathbf{p}} \in \mathbb{R}^m. \quad (15)$$

- I) $\text{rank } \mathbf{K} = m = \hat{m}$. The solution for these basic systems is presented in Theorem 8.
- II) $\text{rank}(\mathbf{K}) < m = \hat{m}$. This system has a solution of dimension $(m - \text{rank } \mathbf{K})$. It can also be solved in a straightforward manner using QR decomposition, for example.
- III) $\text{rank}(\mathbf{K}) = m < \hat{m}$. This system comprises more inequalities than parameters; it is overdetermined, but has full rank. If there is any solution, then the most robust one is

$$\tilde{\mathbf{p}}_{\text{opt}}(q) = q(\mathbf{K}^t \mathbf{K})^{-1} \mathbf{K}^t \mathbf{1}^{\hat{m}}. \quad (16)$$

If $\tilde{\mathbf{p}}_{\text{opt}}(q)$ is not a solution, then the underlying task is ill defined and cannot be realized with this class of CNN's.

Note that in this case, the safety margin may not be equal to q but has to be determined by evaluating (5).

Proof: We project $\mathbf{K}$ into the subspace of dimension $m \times m$ by multiplication with $\mathbf{K}^t$ from the left and solve the new system $\mathbf{K}^t \mathbf{K} \tilde{\mathbf{p}}_{\text{opt}}(q) = q \mathbf{K}^t \mathbf{1}^{\hat{m}}$ yielding a solution in the least squares sense. If $\mathbf{K} \tilde{\mathbf{p}} = q \mathbf{1}^{\hat{m}}$ has an exact solution, then it is $\tilde{\mathbf{p}}_{\text{opt}}$. If there is no exact solution, but the system (15) is not inconsistent, then there is always a point $\tilde{\mathbf{p}}_{\text{opt}}$ with the lowest mean square distance between $\mathbf{K} \tilde{\mathbf{p}}$ and $q \mathbf{1}^{\hat{m}}$.

However, it is not guaranteed that the template $\tilde{\mathbf{p}}_{\text{opt}}(q)$ we get from (16) in fact solves (15); this must be checked. If it does not, then the system is not consistent.

- IV) $\text{rank}(\mathbf{K}) < m < \hat{m}$. In this case, the number of parameters is to be reduced by matrix algebra to achieve $m = \text{rank}(\mathbf{K})$. The system can then be solved using the method proposed for the previous case.
- V) $\text{rank}(\mathbf{K}) \leq \hat{m} < m$. If the number of parameters exceeds the number of inequalities, there is always a solution which has the dimension $(\hat{m} - \text{rank } \mathbf{K})$.

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