Kurt Heutschi, Acoustics I, 2016-10-08

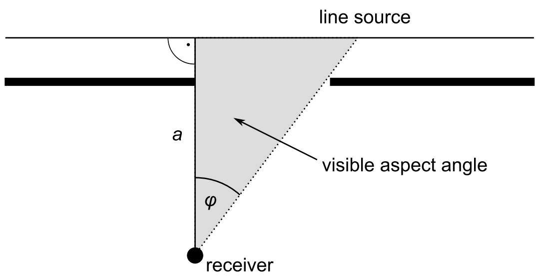
Excercise A1-3: Fundamentals

1. A plane wave with sinusoidal time dependency produces a sound pressure level of 100 dB. Calculate with $Z = \rho c = 420$ Ns/m^3 the particle displacement amplitude for frequencies 100 Hz and 1 kHz.

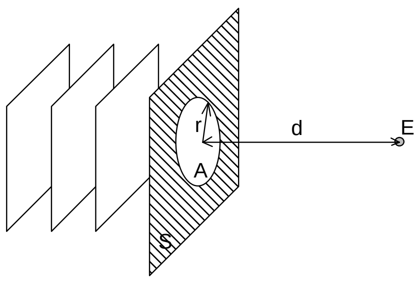
2. The sound pressure level of a machine in a construction hall shall be determined. With the machine switched off, the background noise level is measured as 85 dB. With machine switched on, the total sound pressure level is 88 dB. Calculate the sound pressure level of the machine alone.

3. A person shouts into a cup of water with a volume of 0.2 l. It is assumed that all acoustical energy is converted into heat. How long does the person has to shout to increase the temperature of the water from initial 20°C to 95°C? Note: the heat capacity of water is $\chi = 4.2 \times 10^3$ Jkg^{-1}K^{-1}.

4. A incoherently radiating line source with infinite extension is given (see Figure). A section of length l emits the power $K \times l$. A receiver is located in distance a from the line. Due to shielding the segment of the line source visible for the receiver spans an aspect angle φ. Calculate the RMS value of the sound pressure at the receiver as a function of the angle φ.



5. An arrangement according to the Figure below is given. A plane wave with frequency f travels from left to right and hits the screen S with a circular opening A of radius r. A receiver E is located in distance d from the center of the opening. By variation of the radius r of the opening, the sound pressure in E can be adjusted. Calculate r for an amplification of 6 dB and an attenuation of 10 dB compared to the sound pressure of the plane wave (free field). The concept of Fresnel zones is helpful to solve this problem. It is assumed that the sound field in the opening corresponds to the plane wave sound field. Boundary effects at the border of the opening are neglected.



Solutions to Exercise A1-3

1.
The relation between sound pressure and sound pressure level is: $L_p = 10\log\left(\frac{p_{\mathsf{rms}}^2}{p_0^2}\right)$
Thus a level 100 dB corresponds to a sound pressure $p_{\mathsf{rms}} = 2\ \mathrm{Nm}^{-2}$.
For plane waves the ratio $\frac{p}{v} = \rho c$ is known. The RMS value of the sound particle velocity is $v_{\mathsf{rms}} = 0.0048\ \mathrm{ms}^{-1}$.
The sound particle velocity corresponds to the temporal derivative of the sound particle displacement ζ. Consequently $\zeta = \int v dt$.
With $v = \hat{V}e^{j\omega t}$ follows $\zeta = \hat{V}\frac{1}{j\omega}e^{j\omega t}$. The amplitude of the sound particle displacement is then $\hat{\zeta} = \frac{\sqrt{2}v_{\mathsf{rms}}}{2\pi f}$.
$f = 100$ Hz yields $\hat{\zeta} = 10.7\ \mu$m, $f = 1$ kHz yields $\hat{\zeta} = 1.07\ \mu$m.

2.
It can be assumed that the contributions of the machine and the background noise sum up incoherently at the receiver. Thus the summation holds for the sound pressure squares. Calculating the corresponding difference yields as sound pressure level of the machine alone: $L_{\mathsf{machine}} = 10\log\left(10^{(0.1\cdot 88.0)} - 10^{(0.1\cdot 85.0)}\right) = 85$ dB.

3.
The necessary energy equals heat capacity × mass × temperature change: $\Delta Q = \chi m \Delta T$. This energy corresponds to the product power × time: $\Delta Q = Wt$. From that follows for the time $t = \frac{\chi m \Delta T}{W}$. As maximum power of the human voice a value of $W = 2$ mW can be assumed. With the heat capacity $\chi = 4.2\times 10^3\ \mathrm{Jkg}^{-1}\mathrm{K}^{-1}$, the mass of the water $m = 0.2$ kg and the temperature change $\Delta T = 75$ K the necessary time t calculates as 1 year.

4.
The resulting sound pressure is given as integration over the visible section ($a\tan(\varphi)$) of the line source. In distance r, the sound intensity I that stems from the infinitesimal section dx is given as $I = \frac{K\mathrm{d}x}{4\pi r^2}$. For not too low frequencies f and not too small distances r ($fr > 100$) holds $p_{\mathsf{rms}}^2 = \rho c I$. For the integration can be written:

$$p_{\mathsf{rms}}^2(E) = \int_0^{a\tan(\varphi)} \frac{\rho c K}{4\pi r^2}\mathrm{d}x \text{ and with } r^2 = a^2 + x^2 \rightarrow p_{\mathsf{rms}}^2(E) = \frac{\rho c K}{4\pi}\int_0^{a\tan(\varphi)} \frac{1}{a^2+x^2}\mathrm{d}x = \frac{\rho c K}{a 4\pi}\varphi$$

The proportionality between $p_{\mathsf{rms}}^2(E)$ and the visible aspect angle φ is a very important property of finite line sources.

5.
If the opening of the screen corresponds exactly to the first Fresnel zone, the sound pressure in E doubles compared to the free field case (+6 dB). The radius r of the first Fresnel zone is defined by the condition $r^2 + d^2 = (d + \frac{\lambda}{2})^2 \rightarrow r_{+6} = \sqrt{d\lambda + \frac{\lambda^2}{4}}$ where λ is the wave length, $\lambda = \frac{c}{f}$, c: speed of sound, f: frequency. For smaller radii of the opening, the sound pressure at the receiver decreases. An attenuation of 10 dB relative to free field corresponds to the reduction of the opening area to 1/6 of the area of the first Fresnel zone. For the radius follows: $r_{-10} \approx \frac{1}{\sqrt{6}}\sqrt{d\lambda + \frac{\lambda^2}{4}}$