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Practical course: Loudspeakers

1 Introduction

Loudspeakers convert electrical signals into acoustical ones. The majority of the loudspeakers in use today are based on a moving membrane to excite the sound field. The force to make the membrane move can be produced by different means. Most often one takes advantage of the electro-dynamical converter principle. Hereby the force on a current-carrying conductor generated by a magnetic field is utilized. The sound radiation by the moving membrane depends on its dimension and the surrounding. The radiated power W can be expressed as:

$$W = Q^2 Re[Z_R] \tag{1}$$

with
Q: volume velocity ($Q = S \cdot v_m$ where S: membrane area and v_m: membrane velocity)
$Re[Z_R]$: real part of the radiation impedance $Z_R = p/Q$ where p: sound pressure on the membrane

The radiation impedance $Z_{Ro} = p/v_m$ of a membrane mounted in a infinitely extended wall has real and imaginary parts according to Figure 1 (left). The parameter ka used is the product of the wave number $k = 2\pi/\lambda$ (λ: wave length) and the radius a of the membrane. For small ka values the real part of the radiation impedance is proportional to k^2. For $ka > 2$ the real part is almost constant and approximates the value of the impedance of plane waves. The right hand side of Figure 1 shows the radiation impedance for a free membrane without any surrounding. Here for small ka values the real part increases proportionally to k^4. It is obvious that a free membrane is very inefficient in radiating low frequencies.

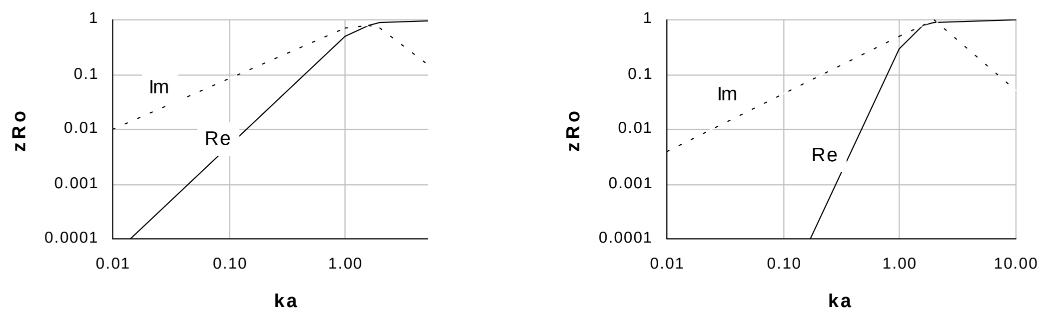

Figure 1: Real- and imaginary part of the radiation impedance Z_{Ro} as a function of ka (k: wave number, a: radius of the membrane). The impedance is shown relative to the free field impedance of plane waves = ρc. Left: membrane mounted in an infinitely extended wall, right: free membrane.

The system *loudspeaker* can be expressed by an analogue electrical circuit as shown in Figure 2. The network elements depend on the following parameters:

M_{AR}, R_{AR} acoustical mass and resistance of the air at the rear side of the membrane (corresponding to real and imaginary parts of the radiation impedance)

m, s mechanical mass of membrane and coil and stiffness of membrane and suspension

R_m mechanical friction losses of the suspension of the membrane

M_{AV}, R_{AV} acoustical mass and resistance of the air at the front side of the membrane (corresponding to the real and imaginary parts of the radiation impedance)

R_E, L_E electrical resistance and inductance of the coil

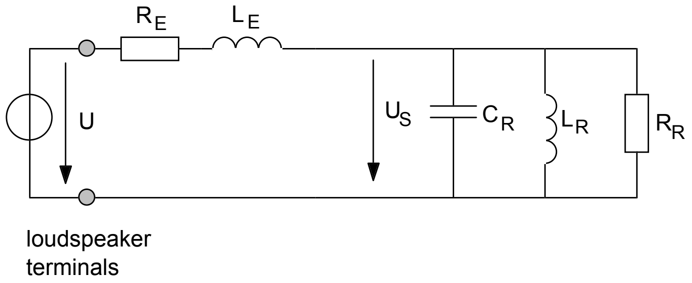

Figure 2: Equivalent electrical network of the dynamic loudspeaker.

The network in Figure 2 represents a resonance system. Hereby the voltage U_S is proportional to the velocity of the membrane. Above resonance, the amplitude of the membrane velocity is proportional to $1/f$. This just compensates for the f^2 dependency of the real part of the radiation impedance if the chassis is mounted in an infinitely extended wall. In consequence, the radiated sound power becomes frequency independent above resonance and below the frequency limit for which $ka < 1$.

The practical realization of an infinitely extended wall (which actually separates the front and the rear side of the membrane) is a cabinet. However the air enclosed in this box acts as an additional spring and will thus increase the resonance frequency.

Loudspeaker chassis are usually characterized by their Thiele-Small parameters [1]. They allow for a comfortable estimate of the properties of the loudspeaker mounted in a box of arbitrary volume.

The radiation of the whole audio frequency range from 20 Hz up to 20 kHz is very challenging. In most cases the frequency range is separated into several bands and fed to specialized chassis. This leads to the concept of loudspeaker systems consisting of 2, 3 or sometimes 4 chassis and corresponding crossover networks (filters).

The reproduction of low frequencies is one of the fundamental challenges in the construction of loudspeakers. A trick to improve the low frequency behavior is the usage of so called bass reflex cabinets. This cabinet type is equipped with an additional opening that is connected to the interior by a tube of distinct length and cross section. The mass of the air in the tube acts together with the compliance of the air in the cabinet as a spring-mass resonator. By appropriate tuning of this resonance the range of operation of the loudspeaker can be extended somewhat towards lower frequencies.

The starting point for this practical course is a prefabricated bass-reflex box with a woofer and a tweeter already mounted. By installing an additional board, the effective box volume can be adjusted at will. In several sessions, some fundamental aspects of sound generation by loudspeakers shall be explored. After that the Thiele-Small parameters of the woofer are to be determined and the optimal cabinet volume will be chosen. Finally the crossover network is designed and put together.

2 Tasks

2.1 Sound pressure amplitude response measurements

For both the woofer and the tweeter, the on-axis sound pressure amplitude response is to be measured in the anechoic chamber. The distance between loudspeaker and microphone shall be chosen as approximately 1.5 m. The woofer measurement shall be performed for the following configurations:

- cabinet with backplane removed

[1] Richard H. Small, Closed-Box Loudspeaker Systems, Part I: Analysis, J. Audio Engineering Society, vol. 20, no. 10 (1972), 798-808.

- closed cabinet without porous filling
- closed cabinet with porous filling
- closed cabinet with open/closed bass reflex opening

The frequency response measurements shall be discussed. It should be investigated to what extent the low frequency end of the measurements is influenced by room resonances as the absorption gets weaker for frequencies below about 80 Hz.

2.2 Measurement of the Thiele-Small parameters of the woofer

From measurements of the amplitude response of the electrical impedance (see Figure 3) the following parameters of the woofer shall be determined:

- resonance frequency f_s
- total Q-factor: Q_{ts}
- compliance equivalent volume V_{AS}
- DC resistance R_E
- inductance of the moving coil L

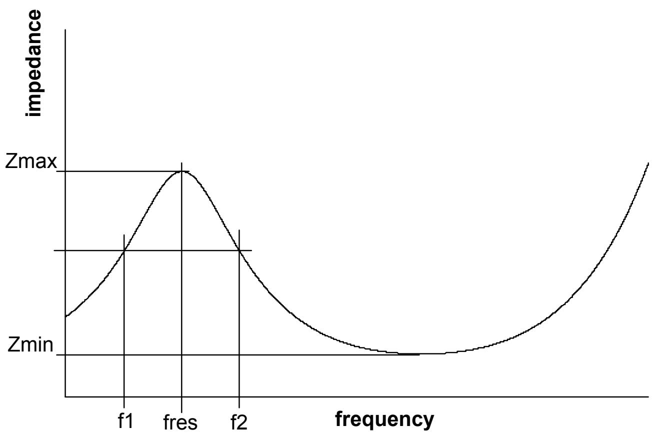

Figure 3: Amplitude response of the electrical impedance of a dynamic loudspeaker.

2.2.1 Impedance measurement

The impedance measurement is performed with a generator that produces sinusoidal voltage signals and an RMS voltmeter. The voltage source is transformed into a current source by insertion of a series resistor of 1 kOhm. As the impedance of the loudspeaker is much smaller, the current is determined by the series resistor and thus the current amplitude is constant. The voltage measured across the loudspeaker terminals is thus proportional to the impedance.

2.2.2 Impedance measurement of the free chassis

As a first step the frequency response of the impedance of the free chassis is measured. For that purpose, the backplane is removed. From this measurement the parameters are deduced:

- resonance frequency $f_s = f_{res}$
- DC resistance $R_E = Z_{min}$

The inductance of the moving coil can be determined at the upper frequency end.

In addition, the following quantities are needed:

- $r_0 = \frac{Z_{max}}{Z_{min}}$
- f_1 and f_2

f_1 and f_2 are the frequencies to the left and the right of the resonance frequency for which the impedance value corresponds to $Z = \sqrt{r_0} Z_{min}$. For further processing, the following quantities are needed:

- $f_{1s} = f_1$
- $f_{2s} = f_2$
- $r_{0s} = r_0$

With these quantities the quality factors can be calculated as:

mechanical Q-factor

$$Q_{ms} = \frac{f_s \sqrt{r_{0s}}}{f_{2s} - f_{1s}} \tag{2}$$

electrical Q-factor

$$Q_{es} = \frac{Q_{ms}}{r_{0s} - 1} \tag{3}$$

total Q-factor

$$Q_{ts} = \frac{Q_{ms} Q_{es}}{Q_{ms} + Q_{es}} \tag{4}$$

2.2.3 Impedance measurement of the chassis mounted in a small volume

For the determination of the last unknown parameter V_{AS}, the impedance measurement has to be repeated with the chassis mounted in a small volume V_t. Note that the cavity should *not* be filled with damping material. Similarly to the first measurement, the quantities

- $f_{ct} = f_{res}$
- $f_{1ct} = f_1$
- $f_{2ct} = f_2$
- $r_{0ct} = r_0$

are calculated. With this follows:

$$Q_{mct} = \frac{f_{ct} \sqrt{r_{0ct}}}{f_{2ct} - f_{1ct}} \tag{5}$$

$$Q_{ect} = \frac{Q_{mct}}{r_{0ct} - 1} \tag{6}$$

and finally

$$V_{AS} = V_t \left(\frac{f_{ct} Q_{ect}}{f_s Q_{es}} - 1 \right) \tag{7}$$

2.3 Optimal cabinet volume

By mounting the woofer chassis in a cabinet of volume V_B, the total quality factor Q_{tc} and the resonance frequency f_c change as follows:

$$Q_{tc} = Q_{ts}\sqrt{1 + \frac{V_{AS}}{V_B}} \tag{8}$$

$$f_c = f_s\sqrt{1 + \frac{V_{AS}}{V_B}} \tag{9}$$

The sound pressure transfer function of the loudspeaker corresponds to a high-pass function with a principle frequency dependency as shown in Figure 4.

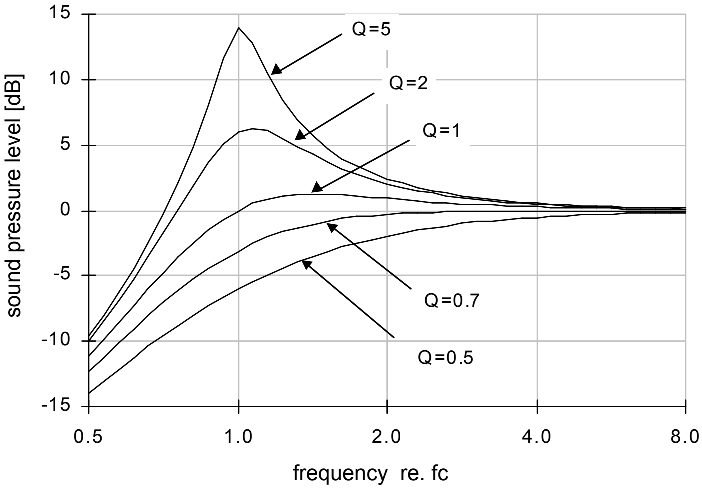

Figure 4: Sound pressure frequency response of the dynamic loudspeaker for different values of the total quality factor Q (abscissa as frequency relative to the resonance f_c).

The optimal cabinet volume is usually determined in such a way that the total quality factor reaches a value between 0.7 and 1.0. It has to be considered that the cabinet is later filled with damping material. Due to the increased thermal conductivity compared to plain air, the acoustical processes are no longer adiabatic but isothermal. This causes the effective cabinet volume to appear about 15% larger than the geometrical one. The desired cabinet volume can be adjusted by inserting the small board at the appropriate position. By adjusting the box volume, the resonance frequency of the bass reflex system is influenced as well. If necessary a compromise between the two requirements has to be chosen.

2.4 Design of the crossover network

Based on the sound pressure frequency response measurements of the two chassis, a suitable crossover frequency f_{sep} has to be chosen. It should be noted that both chassis contribute equally to the sound pressure at f_{sep}. As a pressure doubling corresponds to an amplification of 6 dB, the low-pass filter for the woofer and the high-pass filter for the tweeter should attenuate by 6 dB at f_{sep}. The upper limiting frequency of the low-pass filter must thus lie somewhat lower and the lower limiting frequency of the high-pass filter somewhat higher than f_{sep}.

The filters to be used here are second order passive LC filters (Figure 5).
For a given R (corresponds to the DC-Resistance of the chassis) and a limiting frequency f_g (-3 dB point!) the element values C and L can be calculated according to:

$$C = \frac{\sqrt{2}}{4\pi f_g R} \qquad \text{[Farad]} \tag{10}$$

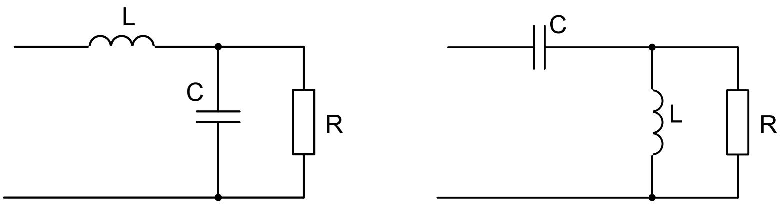

Figure 5: Second order crossover networks: low-pass filter (left) and high-pass filter (right). A resistive termination R is assumed.

$$L = \frac{\sqrt{2}R}{2\pi f_g} \qquad \text{[Henry]} \tag{11}$$

The equations above assume a resistive load with constant impedance over frequency. As already observed, the electrical impedance of the woofer is frequency dependent. Of special importance in this context is the impedance increase towards higher frequencies due to the inductance of the moving coil. This impedance increase can be compensated by insertion of a network as shown in Figure 6.

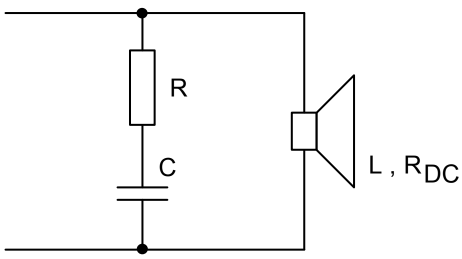

Figure 6: High-frequency compensation of the impedance of the woofer with moving coil inductance L and DC resistance R_{DC}.

The values of R and C are determined with help of the following formulas:

$$R = 1.5R_{DC} \tag{12}$$

$$C = \frac{L}{R^2} \tag{13}$$

The crossover network inclusive the impedance compensation shall be built on an experimenting board. Finally the frequency response of the complete loudspeaker system is measured in the anechoic chamber.

3 Result collection

speaker (A,B,C,D):	

Thiele-Small parameters of the woofer

resonance frequency f_s:	
total Q-factor Q_{ts}:	
compliance equivalent volume V_{AS}:	
DC resistance R_E:	
inductance of the moving coil L:	

Dimensioning of the cabinet

total Q-factor of the woofer in the box Q_{tc}:	
box volume V_B:	
resonance frequency of the woofer in the box f_c:	

Dimensioning of the crossover network

upper limiting frequency low-pass f_{lp}:	
lower limiting frequency high-pass f_{hp}:	