Kurt Heutschi, Acoustics II, 2012-01-09

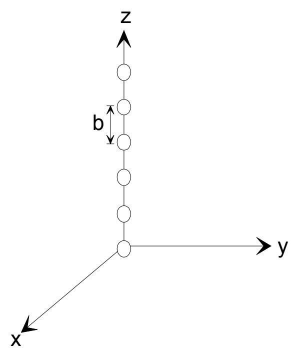
Exercise A2-9: Recordings and Reproduction

1. Intensity stereo generates directional information exclusively by level differences between the two channels. Here it is assumed that during reproduction a source is heard laterally if the inter channel level difference in the recording is 10 dB. The *opening angle* of a recording describes the azimuthal angle that is mapped to the region between the left and right lateral directions.

a) Calculate the opening angle for XY stereophony if the the two microphones form an angle of 120°?
b) Calculate the weighting factor of the lateral signal in an MS configuration with an omni and a figure of eight microphone for the same opening angle as found in a)? (Assumption: both microphone have identical sensitivity)

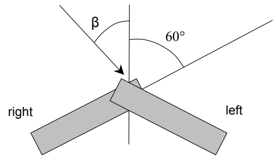
2. In audio recordings the distance of the microphone to the source is often chosen as the critical distance for a good compromise between direct sound and room information. In the critical distance the sound pressure square of the direct sound and the diffuse field are equal. The sound pressure square of the direct sound is inversely proportional to the square of the distance, the diffuse field is constant all over the room. Calculate the distance for equal contribution of direct and diffuse sound if a cardioid microphone is used instead of an omni.

3. Calculate the horizontal (xy plane) and vertical (xz plane) directivity pattern produced by n loudspeakers equally distributed along a straight line. It is assumed that all speakers radiate the same signal. The characteristics shall be determined for the far field only and the speakers can be modeled by omnidirectionally radiating point sources.



Solutions to Exercise A2-9: Recordings and Reproduction

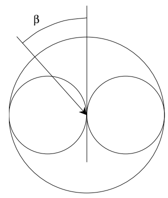
1.
a.



Voltage at the *left* microphone: $U_L = 1 + \cos(60° - \beta)$
Voltage at the *right* microphone: $U_R = 1 + \cos(60° + \beta)$

$$\frac{U_L}{U_R} = 10\text{dB} = 3.15 \rightarrow \beta = 52° \rightarrow \text{ opening angle } = 104° \tag{1}$$

b.



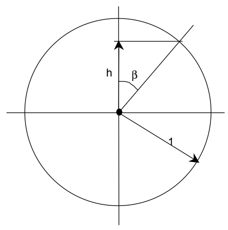
Voltage of the left channel: $U_L = 1 + \mathit{scale} \cdot \sin(\beta)$
Voltage of the right channel: $U_R = 1 - \mathit{scale} \cdot \sin(\beta)$

$$\frac{U_L}{U_R} = 10\text{dB} = 3.15 \text{ mit } \beta = 52° \rightarrow \mathit{scale} = 0.66 \tag{2}$$

The signal of the figure of eight microphone has to be lowered by 3.6 dB with respect to the omni.

2.

The relevant step to the solution is the determination of the signal of the cardioid and the omni microphone produced by the diffuse field. The diffuse field signal captured by the two microphones is found by energetic integration over all possible sound incident directions with corresponding weighting according to the microphone directivity. As the cardioid directivity is rotational symmetric, the integration over the unity sphere can be performed with help of one variable only. Hereby we use the fact that the curved surface area (Mantelfläche) A_L of a spherical segment (Kugelzone) is given as $A_L = 2\pi R\Delta h$, where R: radius of the sphere and Δh: thickness of the spherical segment.



The integration of the squared signal over the unity sphere is

$$U^2 = \int_{-1}^{+1} 2\pi(\Gamma(h))^2 dh \tag{3}$$

where
$\Gamma(h)$: directivity of the microphone as a function of h.

As can be seen in the figure, h corresponds to $\cos(\beta)$. The directivity of the cardioid microphone $U = 0.5(1 + \cos(\beta))$ can be inserted in the integral above as

$$U^2 = \frac{\pi}{2} \int_{-1}^{+1} (1+h)^2 dh = \frac{\pi}{2}\frac{8}{3} \tag{4}$$

The integration for the omnidirectional microphone is

$$U^2 = 2\pi \int_{-1}^{+1} 1 dh = 4\pi \tag{5}$$

The ratio of the squared diffuse field signal at the cardioid microphone and at the omni is 1/3 or -4.8 dB. The diffuse field level at the cardioid microphone is 4.8 dB lower. Thus the distance to the source has to be increased in such a way that the direct is lowered by 4.8 dB as well. This corresponds to a distance factor of 1.7.

3.

Horizontal plane (xy):

The directivity in the horizontal plane is round as in the far field the contributions of all sources superpose with identical phase.

Vertical plane (xz):

The sound pressure at a receiver in the far field is calculated as the phase sensitive superposition of the contributions of all sources. In complex writing this is:

$$\underline{p} = \hat{p} \sum_{i=0}^{n-1} \frac{1}{r_i} e^{j(\omega t - k r_i)} \tag{6}$$

where $\hat{p}$: amplitude, r_i: distance source-receiver, k: wave number $= \frac{2\pi}{\lambda}$.

For a receiver in the far field under an angle ϕ with respect to the normal direction to the line of sources, r_i is given as: $r_i = r - ib\sin(\phi)$.

The amplitude term $\frac{1}{r_i}$ in the far field can be assumed constant. The phase term r_i is

$$\underline{p} = \hat{p} e^{j\omega t} \frac{1}{r} \sum_{i=0}^{n-1} e^{jkib\sin(\phi)} \tag{7}$$

The expression in the sum corresponds to a geometrical series. With the abbreviation $\Delta = \frac{kb\sin(\phi)}{2}$ follows

$$\underline{p} = \hat{p} e^{j\omega t} \frac{1}{r} \sum_{i=0}^{n-1} e^{j2\Delta i} = \hat{p} e^{j\omega t} \frac{1}{r} \frac{1 - e^{j2n\Delta}}{1 - e^{j2\Delta}} = \hat{p} e^{j\omega t} \frac{1}{r} \frac{e^{jn\Delta}\left(e^{-jn\Delta} - e^{jn\Delta}\right)}{e^{j\Delta}\left(e^{-j\Delta} - e^{j\Delta}\right)} = \hat{p} e^{j\omega t} \frac{1}{r} \frac{\sin(n\Delta)}{\sin(\Delta)} e^{j(n-1)\Delta} \tag{8}$$

The directivity manifests in the term

$$\frac{\sin(n\Delta)}{\sin(\Delta)} = \frac{\sin\left(\frac{nb\pi}{\lambda}\sin(\phi)\right)}{\sin\left(\frac{b\pi}{\lambda}\sin(\phi)\right)} \tag{9}$$

For small values of ϕ, the expression tends to 1. For increasing ϕ the denominator increases faster than the nominator. Therefore the expression gets smaller and smaller. The decrease is steeper for larger values of $\frac{nb}{\lambda}$. This signifies that the directivity is very pronounced for large arrays and high frequencies.