Communication and Detection Theory

Signal and Information Processing Laboratory
Institut für Signal- und Informationsverarbeitung

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http://www.isi.ee.ethz.ch/teaching/courses/cdt

Problem 1 *Manipulating Inner Products*

Show that if $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are energy-limited complex signals, then

$$\langle \mathbf{u}+\mathbf{v}, 3\mathbf{u}+\mathbf{v}+\mathsf{i}\mathbf{w}\rangle = 3\,\|\mathbf{u}\|_2^2 + \|\mathbf{v}\|_2^2 + \langle \mathbf{u},\mathbf{v}\rangle + 3\,\langle \mathbf{u},\mathbf{v}\rangle^* - \mathsf{i}\,\langle \mathbf{u},\mathbf{w}\rangle - \mathsf{i}\,\langle \mathbf{v},\mathbf{w}\rangle\,.$$

Problem 2 *Orthogonality to All Signals*

Let $\mathbf{u}$ be an energy-limited signal. Show that

$$\Big(\mathbf{u}\equiv\mathbf{0}\Big) \Leftrightarrow \Big(\langle \mathbf{u},\mathbf{v}\rangle = 0, \quad \mathbf{v}\in\mathcal{L}_2\Big).$$

Problem 3 *Finite-Energy Signals*

Let $\mathbf{x}$ be an energy-limited signal.

(i) Show that, for every $t_0\in\mathbb{R}$, the signal $t\mapsto x(t-t_0)$ must also be energy-limited.

(ii) Show that the reflection of $\mathbf{x}$ is also energy-limited. I.e., show that the signal $\check{\mathbf{x}}$ that maps t to $x(-t)$ is energy-limited.

(iii) How are the energies in $t\mapsto x(t)$, $t\mapsto x(t-t_0)$, and $t\mapsto x(-t)$ related?

Problem 4 *Inner Products of Mirror Images*

Express the inner product $\langle \check{\mathbf{x}},\check{\mathbf{y}}\rangle$ in terms of the inner product $\langle \mathbf{x},\mathbf{y}\rangle$.

Problem 5 *Truncated Polynomials*

Consider the signals $\mathbf{u}\colon t\mapsto (t+2)\,\mathrm{I}\{0\le t\le 1\}$ and $\mathbf{v}\colon t\mapsto (t^2-2t-3)\,\mathrm{I}\{0\le t\le 1\}$. Compute the energies $\|\mathbf{u}\|_2^2$ & $\|\mathbf{v}\|_2^2$ and the inner product $\langle \mathbf{u},\mathbf{v}\rangle$.