Problem 1  \hspace{1cm} \textit{Reflection of Passband Signal}

Let \( x_{\text{PB}} \) and \( y_{\text{PB}} \) be real integrable passband signals that are bandlimited to \( W \) Hz around the carrier frequency \( f_c \). Let \( x_{\text{BB}} \) and \( y_{\text{BB}} \) be their baseband representations.

(i) Express the baseband representation of \( x_{\text{PB}} \) in terms of \( x_{\text{BB}} \).

(ii) Express \( \langle x_{\text{PB}}, y_{\text{PB}} \rangle \) in terms of \( x_{\text{BB}} \) and \( y_{\text{BB}} \).

Problem 2  \hspace{1cm} \textit{Symmetries of the FT}

Let \( x : \mathbb{R} \to \mathbb{C} \) be integrable, and let \( \hat{x} \) be its FT.

(i) Show that if \( x \) is a real signal, then \( \hat{x} \) is conjugate symmetric, i.e., \( \hat{x}(-f) = \hat{x}^*(f) \), for every \( f \in \mathbb{R} \).

(ii) Show that if \( x \) is purely imaginary (i.e., takes on only purely imaginary values), then \( \hat{x} \) is conjugate antisymmetric, i.e., \( \hat{x}(-f) = -\hat{x}^*(f) \), for every \( f \in \mathbb{R} \).

(iii) Show that \( \hat{x} \) can be written uniquely as the sum of a conjugate-symmetric function \( g_{\text{cs}} \) and a conjugate-antisymmetric function \( g_{\text{cas}} \). Express \( g_{\text{cs}} \) & \( g_{\text{cas}} \) in terms of \( \hat{x} \).

Problem 3  \hspace{1cm} \textit{Phase Shift}

Let \( x \) be a real integrable signal that is bandlimited to \( W \) Hz. Let \( f_c \) be larger than \( W \).

(i) Express the baseband representation of the real passband signal

\[
z_{\text{PB}}(t) = x(t) \sin(2\pi f_c t + \phi), \quad t \in \mathbb{R}
\]

in terms of \( x(\cdot) \) and \( \phi \).

(ii) Compute the Fourier Transform of \( z_{\text{PB}} \).
Problem 4

Purely Real and Purely Imaginary Baseband Representations

Let $x_{PB}$ be a real integrable passband signal that is bandlimited to $W$ Hz around the carrier frequency $f_c$, and let $x_{BB}$ be its baseband representation.

(i) Show that $x_{BB}$ is real if, and only if, $\hat{x}_{PB}$ satisfies

$$\hat{x}_{PB}(f_c - \delta) = \hat{x}_{PB}(f_c + \delta), \quad |\delta| \leq \frac{W}{2}.$$ 

(ii) Show that $x_{BB}$ is imaginary if, and only if,

$$\hat{x}_{PB}(f_c - \delta) = -\hat{x}_{PB}(f_c + \delta), \quad |\delta| \leq \frac{W}{2}.$$ 

Problem 5

Symmetry around the Carrier Frequency

Let $x_{PB}$ be a real integrable passband signal that is bandlimited to $W$ Hz around the carrier frequency $f_c$.

(i) Show that $x_{PB}$ can be written in the form

$$x_{PB}(t) = w(t) \cos(2\pi f_c t), \quad t \in \mathbb{R}$$

where $w(\cdot)$ is a real integrable signal that is bandlimited to $W/2$ Hz if, and only if,

$$\hat{x}_{PB}(f_c + \delta) = \hat{x}_{PB}(f_c - \delta), \quad |\delta| \leq \frac{W}{2}.$$ 

(ii) Show that $x_{PB}$ can be written in the form

$$x_{PB}(t) = w(t) \sin(2\pi f_c t), \quad t \in \mathbb{R}$$

for $w(\cdot)$ as above if, and only if,

$$\hat{x}_{PB}(f_c + \delta) = -\hat{x}_{PB}(f_c - \delta), \quad |\delta| \leq \frac{W}{2}.$$ 

Problem 6

Be sure you can justify and derive all the entries in Table 7.1.

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