Communication and Detection Theory

Signal and Information Processing Laboratory
Institut für Signal- und Informationsverarbeitung

Spring Semester 2017

Prof. Dr. A. Lapidoth

Exercise 2 of Februar 28, 2017

http://www.isi.ee.ethz.ch/teaching/courses/cdt

Problem 1 — *Reflection of Passband Signal*

Let $\mathbf{x}_{\text{PB}}$ and $\mathbf{y}_{\text{PB}}$ be real integrable passband signals that are bandlimited to W Hz around the carrier frequency f_{c}. Let $\mathbf{x}_{\text{BB}}$ and $\mathbf{y}_{\text{BB}}$ be their baseband representations.

(i) Express the baseband representation of $\overleftarrow{\mathbf{x}}_{\text{PB}}$ in terms of $\mathbf{x}_{\text{BB}}$.

(ii) Express $\langle \mathbf{x}_{\text{PB}}, \overleftarrow{\mathbf{y}}_{\text{PB}} \rangle$ in terms of $\mathbf{x}_{\text{BB}}$ and $\mathbf{y}_{\text{BB}}$.

Problem 2 — *Symmetries of the FT*

Let $\mathbf{x} \colon \mathbb{R} \to \mathbb{C}$ be integrable, and let $\hat{\mathbf{x}}$ be its FT.

(i) Show that if $\mathbf{x}$ is a real signal, then $\hat{\mathbf{x}}$ is conjugate symmetric, i.e., $\hat{x}(-f) = \hat{x}^*(f)$, for every $f \in \mathbb{R}$.

(ii) Show that if $\mathbf{x}$ is purely imaginary (i.e., takes on only purely imaginary values), then $\hat{\mathbf{x}}$ is conjugate antisymmetric, i.e., $\hat{x}(-f) = -\hat{x}^*(f)$, for every $f \in \mathbb{R}$.

(iii) Show that $\hat{\mathbf{x}}$ can be written uniquely as the sum of a conjugate-symmetric function $\mathbf{g}_{\text{cs}}$ and a conjugate-antisymmetric function $\mathbf{g}_{\text{cas}}$. Express $\mathbf{g}_{\text{cs}}$ & $\mathbf{g}_{\text{cas}}$ in terms of $\hat{\mathbf{x}}$.

Problem 3 — *Phase Shift*

Let $\mathbf{x}$ be a real integrable signal that is bandlimited to W Hz. Let f_{c} be larger than W.

(i) Express the baseband representation of the real passband signal

$$z_{\text{PB}}(t) = x(t)\sin(2\pi f_{\text{c}} t + \phi), \quad t \in \mathbb{R}$$

in terms of $x(\cdot)$ and ϕ.

(ii) Compute the Fourier Transform of $\mathbf{z}_{\text{PB}}$.

Problem 4 — *Purely Real and Purely Imaginary Baseband Representations*

Let $\mathbf{x}_{\text{PB}}$ be a real integrable passband signal that is bandlimited to W Hz around the carrier frequency f_{c}, and let $\mathbf{x}_{\text{BB}}$ be its baseband representation.

(i) Show that $\mathbf{x}_{\text{BB}}$ is real if, and only if, $\hat{\mathbf{x}}_{\text{PB}}$ satisfies

$$\hat{x}_{\text{PB}}(f_{\text{c}} - \delta) = \hat{x}^*_{\text{PB}}(f_{\text{c}} + \delta), \quad |\delta| \leq \frac{W}{2}.$$

(ii) Show that $\mathbf{x}_{\text{BB}}$ is imaginary if, and only if,

$$\hat{x}_{\text{PB}}(f_{\text{c}} - \delta) = -\hat{x}^*_{\text{PB}}(f_{\text{c}} + \delta), \quad |\delta| \leq \frac{W}{2}.$$

Problem 5 — *Symmetry around the Carrier Frequency*

Let $\mathbf{x}_{\text{PB}}$ be a real integrable passband signal that is bandlimited to W Hz around the carrier frequency f_{c}.

(i) Show that $\mathbf{x}_{\text{PB}}$ can be written in the form

$$x_{\text{PB}}(t) = w(t)\cos(2\pi f_{\text{c}} t), \quad t \in \mathbb{R}$$

where $w(\cdot)$ is a real integrable signal that is bandlimited to $W/2$ Hz if, and only if,

$$\hat{x}_{\text{PB}}(f_{\text{c}} + \delta) = \hat{x}^*_{\text{PB}}(f_{\text{c}} - \delta), \quad |\delta| \leq \frac{W}{2}.$$

(ii) Show that $\mathbf{x}_{\text{PB}}$ can be written in the form

$$x_{\text{PB}}(t) = w(t)\sin(2\pi f_{\text{c}} t), \quad t \in \mathbb{R}$$

for $w(\cdot)$ as above if, and only if,

$$\hat{x}_{\text{PB}}(f_{\text{c}} + \delta) = -\hat{x}^*_{\text{PB}}(f_{\text{c}} - \delta), \quad |\delta| \leq \frac{W}{2}.$$

Problem 6 — *Table*

Be sure you can justify and derive all the entries in Table 7.1.