Problem 1

**A Specific Signal**

Let \( x \) be a real energy-limited passband signal that is bandlimited to \( W \) Hz around the carrier frequency \( f_c \). Suppose that all its complex samples are zero except for its zeroth complex sample, which is given by \( 1 + i \). What is \( x \)?

Problem 2

**Multiplying by a Carrier**

Let \( x \) be a real energy-limited signal that is bandlimited to \( W/2 \) Hz, and let \( f_c \) be larger than \( W/2 \). Express the complex samples of \( t \mapsto x(t) \cos(2\pi f_c t) \) in terms of \( x \). Repeat for \( t \mapsto x(t) \sin(2\pi f_c t) \).

Problem 3

**Orthogonal Passband Signals**

Let \( x_{PB} \) and \( y_{PB} \) be real energy-limited passband signals that are bandlimited to \( W \) Hz around the carrier frequency \( f_c \). Under what conditions on their complex samples are they orthogonal?

Problem 4

**The Convolution Revisited**

Let \( x \) and \( y \) be real integrable passband signals that are bandlimited to \( W \) Hz around the carrier frequency \( f_c \). Express the complex samples of \( x \ast y \) in terms of those of \( x \) and \( y \).

Problem 5

**Exploiting Orthogonality**

Let the energy-limited real signals \( \phi_1 \) and \( \phi_2 \) be orthogonal, and let \( A^{(1)} \) and \( A^{(2)} \) be positive constants. Let the waveform \( X \) be given by

\[
X = \left( A^{(1)} X^{(1)} + A^{(2)} X^{(2)} \right) \phi_1 + \left( A^{(1)} X^{(1)} - A^{(2)} X^{(2)} \right) \phi_2,
\]

where \( X^{(1)} \) and \( X^{(2)} \) are unknown real numbers. How can you recover \( X^{(1)} \) and \( X^{(2)} \) from \( X \)?