Problem 1

A Specific Signal

Let $x$ be a real energy-limited passband signal that is bandlimited to $W$ Hz around the carrier frequency $f_c$. Suppose that all its complex samples are zero except for its zeroth complex sample, which is given by $1 + i$. What is $x$?

Problem 2

Multiplying by a Carrier

Let $x$ be a real energy-limited signal that is bandlimited to $W/2$ Hz, and let $f_c$ be larger than $W/2$. Express the complex samples of $t \mapsto x(t)\cos(2\pi f_c t)$ in terms of $x$. Repeat for $t \mapsto x(t)\sin(2\pi f_c t)$.

Problem 3

Orthogonal Passband Signals

Let $x_{PB}$ and $y_{PB}$ be real energy-limited passband signals that are bandlimited to $W$ Hz around the carrier frequency $f_c$. Under what conditions on their complex samples are they orthogonal?

Problem 4

The Convolution Revisited

Let $x$ and $y$ be real integrable passband signals that are bandlimited to $W$ Hz around the carrier frequency $f_c$. Express the complex samples of $x \star y$ in terms of those of $x$ and $y$.

Problem 5

Exploiting Orthogonality

Let the energy-limited real signals $\phi_1$ and $\phi_2$ be orthogonal, and let $A^{(1)}$ and $A^{(2)}$ be positive constants. Let the waveform $X$ be given by

$$X = \left(A^{(1)}X^{(1)} + A^{(2)}X^{(2)}\right)\phi_1 + \left(A^{(1)}X^{(1)} - A^{(2)}X^{(2)}\right)\phi_2,$$

where $X^{(1)}$ and $X^{(2)}$ are unknown real numbers. How can you recover $X^{(1)}$ and $X^{(2)}$ from $X$?