Problem 1  
**A Specific Signal**

Let $x$ be a real energy-limited passband signal that is bandlimited to $W$ Hz around the carrier frequency $f_c$. Suppose that all its complex samples are zero except for its zeroth complex sample, which is given by $1 + i$. What is $x$?

Problem 2  
**Multiplying by a Carrier**

Let $x$ be a real energy-limited signal that is bandlimited to $W/2$ Hz, and let $f_c$ be larger than $W/2$. Express the complex samples of $t \mapsto x(t) \cos(2\pi f_c t)$ in terms of $x$. Repeat for $t \mapsto x(t) \sin(2\pi f_c t)$.

Problem 3  
**Orthogonal Passband Signals**

Let $x_{PB}$ and $y_{PB}$ be real energy-limited passband signals that are bandlimited to $W$ Hz around the carrier frequency $f_c$. Under what conditions on their complex samples are they orthogonal?

Problem 4  
**The Convolution Revisited**

Let $x$ and $y$ be real integrable passband signals that are bandlimited to $W$ Hz around the carrier frequency $f_c$. Express the complex samples of $x * y$ in terms of those of $x$ and $y$.

Problem 5  
**Exploiting Orthogonality**

Let the energy-limited real signals $\phi_1$ and $\phi_2$ be orthogonal, and let $A^{(1)}$ and $A^{(2)}$ be positive constants. Let the waveform $X$ be given by

$$X = \left( A^{(1)} X^{(1)} + A^{(2)} X^{(2)} \right) \phi_1 + \left( A^{(1)} X^{(1)} - A^{(2)} X^{(2)} \right) \phi_2,$$

where $X^{(1)}$ and $X^{(2)}$ are unknown real numbers. How can you recover $X^{(1)}$ and $X^{(2)}$ from $X$?