Communication and Detection Theory

Signal and Information Processing Laboratory
Institut für Signal- und Informationsverarbeitung

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http://www.isi.ee.ethz.ch/teaching/courses/cdt

Problem 1 — *A Specific Signal*

Let $\mathbf{x}$ be a real energy-limited passband signal that is bandlimited to W Hz around the carrier frequency f_c. Suppose that all its complex samples are zero except for its zeroth complex sample, which is given by $1 + i$. What is $\mathbf{x}$?

Problem 2 — *Multiplying by a Carrier*

Let $\mathbf{x}$ be a real energy-limited signal that is bandlimited to $W/2$ Hz, and let f_c be larger than $W/2$. Express the complex samples of $t \mapsto x(t)\cos(2\pi f_c t)$ in terms of $\mathbf{x}$. Repeat for $t \mapsto x(t)\sin(2\pi f_c t)$.

Problem 3 — *Orthogonal Passband Signals*

Let $\mathbf{x}_{\text{PB}}$ and $\mathbf{y}_{\text{PB}}$ be real energy-limited passband signals that are bandlimited to W Hz around the carrier frequency f_c. Under what conditions on their complex samples are they orthogonal?

Problem 4 — *The Convolution Revisited*

Let $\mathbf{x}$ and $\mathbf{y}$ be real integrable passband signals that are bandlimited to W Hz around the carrier frequency f_c. Express the complex samples of $\mathbf{x} \star \mathbf{y}$ in terms of those of $\mathbf{x}$ and $\mathbf{y}$.

Problem 5 — *Exploiting Orthogonality*

Let the energy-limited real signals $\boldsymbol{\phi}_1$ and $\boldsymbol{\phi}_2$ be orthogonal, and let $\mathrm{A}^{(1)}$ and $\mathrm{A}^{(2)}$ be positive constants. Let the waveform $\mathbf{X}$ be given by

$$\mathbf{X} = \left(\mathrm{A}^{(1)} X^{(1)} + \mathrm{A}^{(2)} X^{(2)}\right)\boldsymbol{\phi}_1 + \left(\mathrm{A}^{(1)} X^{(1)} - \mathrm{A}^{(2)} X^{(2)}\right)\boldsymbol{\phi}_2,$$

where $X^{(1)}$ and $X^{(2)}$ are unknown real numbers. How can you recover $X^{(1)}$ and $X^{(2)}$ from $\mathbf{X}$?