Communication and Detection Theory

Signal and Information Processing Laboratory
Institut für Signal- und Informationsverarbeitung

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http://www.isi.ee.ethz.ch/teaching/courses/cdt

Problem 1 — *Passband Signaling*

Let $f_0, \mathsf{T}_s > 0$ be fixed.

(i) Show that a signal $\mathbf{x}$ is a Nyquist Pulse of parameter T_s if, and only if, the signal $t \mapsto e^{i2\pi f_0 t}\, x(t)$ is such a pulse.

(ii) Show that if $\mathbf{x}$ is a Nyquist Pulse of parameter T_s, then so is $t \mapsto \cos(2\pi f_0 t)\, x(t)$.

(iii) If $t \mapsto \cos(2\pi f_0 t)\, x(t)$ is a Nyquist Pulse of parameter T_s, must $\mathbf{x}$ also be one?

Problem 2 — *The Self-Similarity Function of a Delayed Signal*

Let $\mathbf{u}$ be an energy-limited signal, and let the signal $\mathbf{v}$ be given by $\mathbf{v}\colon t \mapsto u(t - t_0)$ for some $t_0 \in \mathbb{R}$. Express the self-similarity function of $\mathbf{v}$ in terms of the self-similarity of $\mathbf{u}$ and t_0.

Problem 3 — *The Self-Similarity Function of a Frequency Shifted Signal*

Let $\mathbf{u}$ be an energy-limited complex signal, and let the signal $\mathbf{v}$ be given by $\mathbf{v}\colon t \mapsto u(t)\, e^{i2\pi f_0 t}$ for some $f_0 \in \mathbb{R}$. Express the self-similarity function of $\mathbf{v}$ in terms of f_0 and the self-similarity function of $\mathbf{u}$.

Problem 4 — *Relaxing the Orthonormality Condition*

What is the minimal bandwidth of an energy-limited signal whose time shifts by even multiples of T_s are orthonormal? What is the minimal bandwidth of an energy-limited signal whose time shifts by odd multiples of T_s are orthonormal?

Problem 5 — *A Specific Signal*

Let $\mathbf{p}$ be the complex energy-limited bandlimited signal whose FT $\hat{\mathbf{p}}$ is given by

$$\hat{p}(f) = \mathsf{T}_s\big(1 - |\mathsf{T}_s f - 1|\big)\, \mathrm{I}\Big\{0 \le f \le \frac{2}{\mathsf{T}_s}\Big\}, \quad f \in \mathbb{R}.$$

(i) Plot $\hat{p}(\cdot)$.

(ii) Is $p(\cdot)$ a Nyquist Pulse of parameter $\mathsf{T_s}$?

(iii) Is the real part of $p(\cdot)$ a Nyquist Pulse of parameter $\mathsf{T_s}$?

(iv) What about the imaginary part of $p(\cdot)$?

Problem 6 ***Mapping a Discrete-Time Stationary SP***

Let $\bigl(X_\nu\bigr)$ be a stationary discrete-time SP, and let $\mathbf{g}\colon \mathbb{R} \to \mathbb{R}$ be some arbitrary (Borel measurable) function. For every $\nu \in \mathbb{Z}$, let $Y_\nu = g(X_\nu)$. Prove that the discrete-time SP $\bigl(Y_\nu\bigr)$ is stationary.

Problem 7 ***Mapping a Discrete-Time WSS SP***

Let $\bigl(X_\nu\bigr)$ be a WSS discrete-time SP, and let $\mathbf{g}\colon \mathbb{R} \to \mathbb{R}$ be some arbitrary (Borel measurable) bounded function. For every $\nu \in \mathbb{Z}$, let $Y_\nu = g(X_\nu)$. Must the SP $\bigl(Y_\nu\bigr)$ be WSS?