Communication and Detection Theory

Signal and Information Processing Laboratory
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http://www.isi.ee.ethz.ch/teaching/courses/cdt

Problem 1 — *Superimposing Independent Transmissions*

Let the two PAM signals $\bigl(X^{(1)}(t)\bigr)$ and $\bigl(X^{(2)}(t)\bigr)$ be given at every epoch $t \in \mathbb{R}$ by

$$X^{(1)}(t) = \mathsf{A}^{(1)} \sum_{\ell=-\infty}^{\infty} X_\ell^{(1)} g^{(1)}(t - \ell \mathsf{T_s}), \quad X^{(2)}(t) = \mathsf{A}^{(2)} \sum_{\ell=-\infty}^{\infty} X_\ell^{(2)} g^{(2)}(t - \ell \mathsf{T_s}),$$

where the zero-mean real symbols $\bigl(X_\ell^{(1)}\bigr)$ are generated from the data bits $\bigl(D_j^{(1)}\bigr)$ and the zero-mean real symbols $\bigl(X_\ell^{(2)}\bigr)$ from $\bigl(D_j^{(2)}\bigr)$. Assume that the bit streams $\bigl(D_j^{(1)}\bigr)$ and $\bigl(D_j^{(2)}\bigr)$ are independent and that $\bigl(X^{(1)}(t)\bigr)$ and $\bigl(X^{(2)}(t)\bigr)$ are of powers $\mathsf{P}^{(1)}$ and $\mathsf{P}^{(2)}$. Find the power in the sum of $\bigl(X^{(1)}(t)\bigr)$ and $\bigl(X^{(2)}(t)\bigr)$.

Problem 2 — *The Minimum Distance of a Constellation and Power*

Consider the PAM signal (14.47) where the time shifts of the pulse shape ϕ by integer multiples of $\mathsf{T_s}$ are orthonormal, and where the symbols $\bigl(X_\ell\bigr)$ are IID and uniformly distributed over the set $\bigl\{\pm\frac{d}{2}, \pm\frac{3d}{2}, \ldots, \pm(2\nu-1)\frac{d}{2}\bigr\}$. Relate the power in $X(\cdot)$ to the minimum distance d and the constant A.

Problem 3 — *PAM with Nonorthogonal Pulses*

Let the IID random bits $\bigl(D_j,\ j \in \mathbb{Z}\bigr)$ be modulated using PAM with the pulse shape $\mathbf{g}\colon t \mapsto \mathrm{I}\{|t| \leq \mathsf{T_s}\}$ and the repetition block encoding map $0 \mapsto (+1, +1)$ and $1 \mapsto (-1, -1)$. Compute the average transmitted power.

Problem 4 — *Non-IID Data Bits*

Expression (14.37) for the power in bi-infinite block mode was derived under the assumption that the data bits are IID. Show that it need not otherwise hold.

Problem 5 — *The Power in Nonorthogonal PAM*

Consider the PAM signal (14.23) with the pulse shape $\mathbf{g}\colon t \mapsto \mathrm{I}\{|t| \leq \mathsf{T_s}\}$.

(i) Compute the power in $X(\cdot)$ when $\bigl(X_\ell\bigr)$ are IID of zero-mean and unit-variance.

(ii) Repeat when $\bigl(X_\ell\bigr)$ is a zero-mean WSS SP of autocovariance function

$$\mathsf{K}_{XX}(m) = \begin{cases} 1 & m = 0 \\ \frac{1}{2} & |m| = 1 \\ 0 & \text{otherwise} \end{cases}, \qquad m \in \mathbb{Z}.$$

Note that in both parts $\mathsf{E}[X_\ell] = 0$ and $\mathsf{E}\bigl[X_\ell^2\bigr] = 1$.

Problem 6 — *Pre-Encoding*

Rather than applying the mapping $\textbf{enc}\colon \{0,1\}^{\mathsf{K}} \to \mathbb{R}^{\mathsf{N}}$ to the IID random bits $D_1, \ldots, D_{\mathsf{K}}$ directly, we first map the data bits using a one-to-one mapping $\boldsymbol{\phi}\colon \{0,1\}^{\mathsf{K}} \to \{0,1\}^{\mathsf{K}}$ to $D'_1, \ldots, D'_{\mathsf{K}}$, and we then map $D'_1, \ldots, D'_{\mathsf{K}}$ using $\textbf{enc}$ to $X_1, \ldots, X_{\mathsf{N}}$. Does this change the transmitted energy?