

# Communication and Detection Theory

Signal and Information  
Processing Laboratory

Institut für Signal- und  
Informationsverarbeitung



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Prof. Dr. A. Lapidoth

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<http://www.isi.ee.ethz.ch/teaching/courses/cdt>

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### Problem 1

### *Scaling a SP*

Let  $(Y(t))$  be the result of scaling the SP  $(X(t))$  by the real number  $\alpha$ . Thus,  $Y(t) = \alpha X(t)$  for every epoch  $t \in \mathbb{R}$ . Show that if  $(X(t))$  is of operational PSD  $S_{XX}$ , then  $(Y(t))$  is of operational PSD  $f \mapsto \alpha^2 S_{XX}(f)$ .

### Problem 2

### *The Operational PSD of a Sum of Independent SPs*

Intuition suggests that if  $(X(t))$  and  $(Y(t))$  are centered independent stochastic processes of operational PSDs  $S_{XX}$  and  $S_{YY}$ , then their sum should be of operational PSD  $f \mapsto S_{XX}(f) + S_{YY}(f)$ . Explain why.

### Problem 3

### *Operational PSD of a Deterministic SP*

Let  $(X(t))$  be deterministically equal to the energy-limited signal  $\mathbf{g}: \mathbb{R} \rightarrow \mathbb{R}$  in the sense that, at every epoch  $t \in \mathbb{R}$ , the RV  $X(t)$  is deterministically equal to  $g(t)$ . Find the operational PSD of  $(X(t))$ .

### Problem 4

### *Stretching Time*

Let  $(X(t))$  be of operational PSD  $S_{XX}$ , and let  $a > 0$  be fixed. Define the SP  $(Y(t))$  at every epoch  $t \in \mathbb{R}$  as  $Y(t) = X(t/a)$ . Show that  $(Y(t))$  is of operational PSD  $f \mapsto a S_{XX}(af)$ .

### Problem 5

### *The Operational PSD of PAM*

Let  $(X_\ell, \ell \in \mathbb{Z})$  be IID with  $X_\ell$  taking on the values  $\pm 1$  equiprobably. Let

$$g(t) = \mathbb{I}\left\{|t| \leq \frac{T_s}{2}\right\}, \quad t \in \mathbb{R},$$

$$X(t) = A \sum_{\ell=-\infty}^{\infty} X_\ell g(t - \ell T_s), \quad t \in \mathbb{R},$$

where  $A, T_s > 0$  are deterministic.

- (i) Plot a sample function of  $\mathbf{X}$  for a realization of  $(X_\ell, \ell \in \mathbb{Z})$  of your choice.

(ii) Compute the operational PSD of  $\mathbf{X}$ .

(iii) Repeat Parts (i) and (ii) for

$$\tilde{X}(t) = A \sum_{\ell=-\infty}^{\infty} X_{\ell} g(t - 2\ell T_s), \quad t \in \mathbb{R}.$$

(iv) How do the operational PSDs of  $\mathbf{X}$  and  $\tilde{\mathbf{X}}$  compare?

### Problem 6

### *The Operational PSD and Block Codes*

PAM is used in block-mode in conjunction with the (1,2) binary-to-reals block encoder

$$0 \mapsto (+1, -1), \quad 1 \mapsto (-1, +1)$$

to transmit IID random bits. The pulse shape  $g(\cdot)$  satisfies the decay condition (14.17). Compute the power and operational PSD of the signal.