Problem 1  
**Scaling a SP**
Let \((Y(t))\) be the result of scaling the SP \((X(t))\) by the real number \(\alpha\). Thus, \(Y(t) = \alpha X(t)\) for every epoch \(t \in \mathbb{R}\). Show that if \((X(t))\) is of operational PSD \(S_{XX}\), then \((Y(t))\) is of operational PSD \(f \mapsto \alpha^2 S_{XX}(f)\).

Problem 2  
**The Operational PSD of a Sum of Independent SPs**
Intuition suggests that if \((X(t))\) and \((Y(t))\) are centered independent stochastic processes of operational PSDs \(S_{XX}\) and \(S_{YY}\), then their sum should be of operational PSD \(f \mapsto S_{XX}(f) + S_{YY}(f)\). Explain why.

Problem 3  
**Operational PSD of a Deterministic SP**
Let \((X(t))\) be deterministically equal to the energy-limited signal \(g: \mathbb{R} \to \mathbb{R}\) in the sense that, at every epoch \(t \in \mathbb{R}\), the RV \(X(t)\) is deterministically equal to \(g(t)\). Find the operational PSD of \((X(t))\).

Problem 4  
**Stretching Time**
Let \((X(t))\) be of operational PSD \(S_{XX}\), and let \(a > 0\) be fixed. Define the SP \((Y(t))\) at every epoch \(t \in \mathbb{R}\) as \(Y(t) = X(t/a)\). Show that \((Y(t))\) is of operational PSD \(f \mapsto \alpha S_{XX}(af)\).

Problem 5  
**The Operational PSD of PAM**
Let \((X_\ell, \ell \in \mathbb{Z})\) be IID with \(X_\ell\) taking on the values \(\pm 1\) equiprobably. Let
\[
g(t) = I\{|t| \leq \frac{T_s}{2}\}, \quad t \in \mathbb{R},
\]
\[
X(t) = A \sum_{\ell=-\infty}^{\infty} X_\ell g(t - \ell T_s), \quad t \in \mathbb{R},
\]
where \(A, T_s > 0\) are deterministic.

(i) Plot a sample function of \(X\) for a realization of \((X_\ell, \ell \in \mathbb{Z})\) of your choice.
(ii) Compute the operational PSD of $X$.

(iii) Repeat Parts (i) and (ii) for

$$
\tilde{X}(t) = A \sum_{\ell=-\infty}^{\infty} X_\ell g(t - 2\ell T_s), \quad t \in \mathbb{R}.
$$

(iv) How do the operational PSDs of $X$ and $\tilde{X}$ compare?

Problem 6

The Operational PSD and Block Codes

PAM is used in block-mode in conjunction with the $(1,2)$ binary-to-reals block encoder

$$
0 \mapsto (+1, -1), \quad 1 \mapsto (-1, +1)
$$

to transmit IID random bits. The pulse shape $g(\cdot)$ satisfies the decay condition (14.17). Compute the power and operational PSD of the signal.