Communication and Detection Theory

Signal and Information Processing Laboratory
Institut für Signal- und Informationsverarbeitung

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http://www.isi.ee.ethz.ch/teaching/courses/cdt

Problem 1 — *How General is QAM?*

Under what conditions on A, f_c, ϕ, W, and $\mathsf{T_s}$ can we view the signal

$$t \mapsto \mathsf{A}\,\mathrm{Re}\left(e^{\mathrm{i}(2\pi f_c t+\phi)} \sum_{\ell=1}^{n} C_\ell \,\mathrm{sinc}\big(W(t-\ell \mathsf{T_s})\big)\right)$$

as a QAM signal?

Problem 2 — *Transmission Rate, Encoder Rate, and Bandwidth*

Data bits are to be transmitted at rate $\mathsf{R_b}$ bits per second using QAM with a pulse shape ϕ satisfying the orthonormality condition (16.11).

(i) Let W be the allotted bandwidth around the carrier frequency. What is the minimal constellation size required for the data bits to be reliably communicated in the absence of noise?

(ii) Repeat Part (i) if you are required to use a pulse shape of excess bandwidth 15% or more.

Problem 3 — *Synthesis of 16-QAM*

Let $X_1(\cdot)$ and $X_2(\cdot)$ be 4-QAM (QPSK) signals that are given for every $t \in \mathbb{R}$ by

$$X_\nu(t) = 2\,\mathrm{Re}\left(\mathsf{A} \sum_{\ell=1}^{n} C_\ell^{(\nu)}\, g(t-\ell \mathsf{T_s})\, e^{\mathrm{i}2\pi f_c t}\right), \quad \nu = 1, 2,$$

where the symbols $\big(C_\ell^{(\nu)}\big)$ take on the values $\pm 1 \pm \mathrm{i}$. Show that for the right choice of the constant $\alpha \in \mathbb{R}$, the signal

$$X(t) = \alpha X_1(t) + X_2(t), \quad t \in \mathbb{R}$$

can be viewed as a 16-QAM signal with a square constellation.

Problem 4 — *Phase Imprecision*

Consider QAM with a real pulse shape and a receiver that performs a conversion to baseband followed by matched filtering (Section 16.8.1). Write an expression for the output of the receiver if its oscillator is at the right frequency but lags the phase of the transmitter's oscillator by $\Delta\phi$.

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Problem 5 ***The Distribution of*** $\mathrm{Re}(Z)$ ***and*** $|Z|$

Let the CRV Z be uniformly distributed over the unit disc $\{z \in \mathbb{C} : |z| \leq 1\}$.

(i) What is the density of its real part $\mathrm{Re}(Z)$?

(ii) What is the density of its magnitude $|Z|$?

Problem 6 ***Product of Proper CRVs***

Show that the product of two independent complex random variables is proper whenever one is proper and the other is of finite variance.

Problem 7 ***Reversing the Direction of Time***

Let K_{ZZ} be the autocovariance function of some discrete-time WSS CSP $\big(Z_\nu\big)$. For every $\nu \in \mathbb{Z}$ define $Y_\nu = Z_{-\nu}$. Show that the time-reversed CSP $\big(Y_\nu\big)$ is also a WSS CSP, and express its autocovariance function K_{YY} in terms of K_{ZZ}.

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