Communication and Detection Theory

Signal and Information Processing Laboratory
Institut für Signal- und Informationsverarbeitung

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Prof. Dr. A. Lapidoth

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http://www.isi.ee.ethz.ch/teaching/courses/cdt

The questions about the operational PSD are optional.

Problem 1 — $\pi/4$-*QPSK*

In QPSK or 4-QAM the data bits are mapped to complex symbols (C_ℓ) which take value in the set $\{\pm 1 \pm \mathrm{i}\}$ and which are then transmitted using the signal $(X(t))$ defined in (18.24). Consider now $\pi/4$-QPSK where, prior to transmission, the complex symbols (C_ℓ) are rotated to form the complex symbols

$$\tilde{C}_\ell = \alpha^\ell \, C_\ell, \quad \ell \in \mathbb{Z},$$

where $\alpha = e^{\mathrm{i}\pi/4}$. The transmitted signal is then

$$2\mathsf{A}\,\mathrm{Re}\Bigl(\sum_{\ell=-\infty}^{\infty} \tilde{C}_\ell \, g(t - \ell \mathsf{T}_\mathrm{s}) \, e^{\mathrm{i}2\pi f_\mathrm{c} t}\Bigr), \quad t \in \mathbb{R}.$$

Compute the power and the operational PSD of the $\pi/4$-QPSK signal when (C_ℓ) is a zero-mean WSS CSP of autocovariance function K_{CC}. Compare the power and operational PSD of $\pi/4$-QPSK with those of QPSK. How do they compare when the symbols (C_ℓ) are IID?
Hint: See Exercise 17.22.

Problem 2 — *The Power in the In-Phase and Quadrature Components*

Consider the setup of Theorem 18.3.1 with the additional assumptions that the real part (and hence, by Exercise 17.17, also the imaginary part) of $(C_\ell, \ \ell \in \mathbb{Z})$ is WSS and that the pulse shape $\mathbf{g}$ is real. Compute the power in each of the signals

$$t \mapsto 2\mathsf{A} \sum_{\ell=-\infty}^{\infty} \mathrm{Re}(C_\ell) \, g(t - \ell \mathsf{T}_\mathrm{s}) \cos(2\pi f_\mathrm{c} t),$$
$$t \mapsto -2\mathsf{A} \sum_{\ell=-\infty}^{\infty} \mathrm{Im}(C_\ell) \, g(t - \ell \mathsf{T}_\mathrm{s}) \sin(2\pi f_\mathrm{c} t),$$

and show that these powers add up to the power in $X(\cdot)$. Give an intuitive explanation for this result. Do you expect a similar result for the operational PSD?
Hint: To compute their power, express the signals as QAM signals and use Theorem 18.3.1.

Problem 3 — *Sums of Independent Gaussians*

Let $X_1 \sim \mathcal{N}\big(0,\sigma_1^2\big)$ and $X_2 \sim \mathcal{N}\big(0,\sigma_2^2\big)$ be independent. Convolve their densities to show that $X_1 + X_2$ is Gaussian.

Problem 4 — *Computing Probabilities*

Let $X \sim \mathcal{N}(1,3)$ and $Y \sim \mathcal{N}(-2,4)$ be independent. Express the probabilities $\Pr[X \leq 2]$ and $\Pr[2X + 3Y > -2]$ using the $\mathcal{Q}$-function with nonnegative arguments.

Problem 5 — *Bounds on the $\mathcal{Q}$-Function*

Prove (19.19). We suggest changing the integration variable in (19.9) to $\zeta \triangleq \xi - \alpha$ and then proving (19.19) using the inequality

$$1 - \frac{\zeta^2}{2} \leq \exp\left(-\frac{\zeta^2}{2}\right) \leq 1, \quad \xi \in \mathbb{R}.$$