Problem 1: Constructing a SP from a RV

Let $W$ be a standard Gaussian RV. Define the continuous-time SP $(X(t))$ by

$$X(t) = e^{-|t|} W, \quad t \in \mathbb{R}. $$

(i) Is $(X(t))$ a stationary SP?
(ii) Is $(X(t))$ a Gaussian SP?

Problem 2: Delaying and Adding

Let $(X(t))$ be a stationary Gaussian SP of mean $\mu_x$ and autocovariance function $K_{XX}$. Define

$$Y(t) = X(t) + X(t - t_D), \quad t \in \mathbb{R},$$

where $t_D \in \mathbb{R}$ is deterministic.

(i) Is $(Y(t))$ a Gaussian SP?
(ii) Compute the mean and the autocovariance function of $(Y(t))$.
(iii) Is $(Y(t))$ stationary?

Problem 3: Random Variables and Stochastic Processes

Let the random variables $X$ and $Y$ be IID $\mathcal{N}(0, \sigma^2)$, and let

$$Z(t) = X \cos(2\pi t) + Y \sin(2\pi t), \quad t \in \mathbb{R}. $$

(i) Is $Z(0.2)$ Gaussian?
(ii) Is $(Z(t))$ a Gaussian SP?
(iii) Is it stationary?
Problem 4  

**Classifying Stochastic Processes**

Let \((X(t))\) and \((Y(t))\) be independent centered stationary Gaussian stochastic processes of unit variance and autocovariance functions \(K_{XX}\) and \(K_{YY}\). Define the stochastic processes \((S(t)), (T(t)), (U(t)), (V(t)),\) and \((W(t))\) at every \(t \in \mathbb{R}\) as:

\[
\begin{align*}
S(t) &= X(t) + Y(t + \tau_1), \\
T(t) &= X(t) Y(t + \tau_2), \\
U(t) &= X(t) + X(t + \tau_3), \\
V(t) &= X(t) X(t + \tau_4), \\
W(t) &= X(t) + X(-t),
\end{align*}
\]

where \(\tau_1, \tau_2, \tau_3, \tau_4 \in \mathbb{R}\) are deterministic. Which of these stochastic processes is Gaussian? Which is WSS? Which is stationary?

Problem 5  

**A Linear Functional of a Gaussian SP**

Let \((X(t), t \in \mathbb{R})\) be a measurable stationary Gaussian SP of mean 2 and of autocovariance function \(K_{XX}: \tau \mapsto \exp(-|\tau|)\). Compute

\[
\Pr\left[ \int_0^2 X(t) \, dt \geq 2 \right].
\]