Communication and Detection Theory

Signal and Information Processing Laboratory
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http://www.isi.ee.ethz.ch/teaching/courses/cdt

Problem 1 — *Constructing a SP from a RV*

Let W be a standard Gaussian RV. Define the continuous-time SP $\big(X(t)\big)$ by

$$X(t) = e^{-|t|}\, W, \quad t \in \mathbb{R}.$$

(i) Is $\big(X(t)\big)$ a stationary SP?

(ii) Is $\big(X(t)\big)$ a Gaussian SP?

Problem 2 — *Delaying and Adding*

Let $\big(X(t)\big)$ be a stationary Gaussian SP of mean μ_x and autocovariance function K_{XX}. Define

$$Y(t) = X(t) + X(t - t_{\mathrm{D}}), \quad t \in \mathbb{R},$$

where $t_{\mathrm{D}} \in \mathbb{R}$ is deterministic.

(i) Is $\big(Y(t)\big)$ a Gaussian SP?

(ii) Compute the mean and the autocovariance function of $\big(Y(t)\big)$.

(iii) Is $\big(Y(t)\big)$ stationary?

Problem 3 — *Random Variables and Stochastic Processes*

Let the random variables X and Y be IID $\mathcal{N}\big(0, \sigma^2\big)$, and let

$$Z(t) = X\cos(2\pi t) + Y\sin(2\pi t), \quad t \in \mathbb{R}.$$

(i) Is $Z(0.2)$ Gaussian?

(ii) Is $\big(Z(t)\big)$ a Gaussian SP?

(iii) Is it stationary?

Problem 4 *Classifying Stochastic Processes*

Let $\big(X(t)\big)$ and $\big(Y(t)\big)$ be independent centered stationary Gaussian stochastic processes of unit variance and autocovariance functions K_{XX} and K_{YY}. Define the stochastic processes $\big(S(t)\big)$, $\big(T(t)\big)$, $\big(U(t)\big)$, $\big(V(t)\big)$, and $\big(W(t)\big)$ at every $t \in \mathbb{R}$ as

$$\begin{aligned} S(t) &= X(t) + Y(t+\tau_1), & T(t) &= X(t)\, Y(t+\tau_2), \\ U(t) &= X(t) + X(t+\tau_3), & V(t) &= X(t)\, X(t+\tau_4), \\ W(t) &= X(t) + X(-t), \end{aligned}$$

where $\tau_1, \tau_2, \tau_3, \tau_4 \in \mathbb{R}$ are deterministic. Which of these stochastic processes is Gaussian? Which is WSS? Which is stationary?

Problem 5 *A Linear Functional of a Gaussian SP*

Let $\big(X(t)\big)$ be a measurable stationary Gaussian SP of mean 2 and of autocovariance function $\mathsf{K}_{XX} \colon \tau \mapsto \exp\left(-|\tau|\right)$. Compute

$$\Pr\left[\int_0^2 X(t)\, \mathrm{d}t \geq 2\right].$$