Communication and Detection Theory

Signal and Information Processing Laboratory
Institut für Signal- und Informationsverarbeitung

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http://www.isi.ee.ethz.ch/teaching/courses/cdt

Problem 1 ***Reading Assignment***

Read Section 26.9.

Problem 2 ***Signaling in White Gaussian Noise***

Let the RV M take value in the set $\mathcal{M} = \{1, 2, 3, 4\}$ uniformly. Conditional on $M = m$, the observed waveform $\bigl(Y(t)\bigr)$ is given at every time $t \in \mathbb{R}$ by $s_m(t) + N(t)$, where the signals $\mathbf{s}_1, \mathbf{s}_2, \mathbf{s}_3, \mathbf{s}_4$ are given by

$$s_1(t) = \mathsf{A}\,\mathrm{I}\{0 \le t \le \mathsf{T}\}, \qquad s_2(t) = \mathsf{A}\,\mathrm{I}\{0 \le t \le \mathsf{T}/2\} - \mathsf{A}\,\mathrm{I}\{\mathsf{T}/2 < t \le \mathsf{T}\},$$
$$s_3(t) = 2\mathsf{A}\,\mathrm{I}\{0 \le t \le \mathsf{T}/2\}, \quad s_4(t) = -\mathsf{A}\,\mathrm{I}\{0 \le t \le \mathsf{T}/2\} + \mathsf{A}\,\mathrm{I}\{\mathsf{T}/2 < t \le \mathsf{T}\},$$

and where $\bigl(N(t)\bigr)$ is WGN of double-sided PSD $\mathsf{N}_0/2$ over the bandwidth of interest. (Ignore the fact that the signals are not bandlimited.)

(i) Derive the MAP rule for guessing M based on $\bigl(Y(t)\bigr)$.

(ii) Use the Union-of-Events Bound to upper-bound $p_{\text{MAP}}(\text{error}|M = 3)$. Are all the terms in the bound needed?

(iii) Compute $p_{\text{MAP}}(\text{error}|M = 3)$ exactly.

(iv) Show that by subtracting a waveform $\mathbf{s}_*$ from each of the signals $\mathbf{s}_1, \mathbf{s}_2, \mathbf{s}_3, \mathbf{s}_4$, we can reduce the average transmitted energy without degrading performance. What waveform $\mathbf{s}_*$ should be subtracted to minimize the transmitted energy?

Problem 3 ***Mismatched Decoding of Antipodal Signaling***

Let the received waveform $\bigl(Y(t)\bigr)$ be given at every $t \in \mathbb{R}$ by $(1 - 2H)\,s(t) + N(t)$, where $\mathbf{s}$ is an integrable signal that is bandlimited to W Hz, $\bigl(N(t)\bigr)$ is WGN of double-sided PSD $\mathsf{N}_0/2$ with respect to the bandwidth W, and H takes on the values 0 and 1 equiprobably and independently of $\bigl(N(t)\bigr)$. Let $\mathbf{s}'$ be an integrable signal that is bandlimited to W Hz. A suboptimal detector feeds the received waveform to a matched filter for $\mathbf{s}'$ and guesses according to the filter's time-0 output: if it is positive, it guesses "$H = 0$," and if it is negative, it guesses "$H = 1$." Express this detector's probability of error in terms of $\mathbf{s}$, $\mathbf{s}'$, and N_0.

Problem 4 ***Imperfect Automatic Gain Control***

Let the received signal $\big(Y(t)\big)$ be given by

$$Y(t) = \mathsf{A} X\, s(t) + N(t), \quad t \in \mathbb{R},$$

where $\mathsf{A} > 0$ is some deterministic positive constant, X is a RV that takes value in the set $\{-3, -1, +1, +3\}$ uniformly, $\mathbf{s}$ is an integrable signal that is bandlimited to W Hz, and $\big(N(t)\big)$ is WGN of double-sided PSD $\mathsf{N}_0/2$ with respect to the bandwidth W.

(i) Find an optimal rule for guessing X based on $\big(Y(t)\big)$.

(ii) Using the $\mathcal{Q}$-function compute the optimal probability of error.

(iii) Suppose you use the rule you have found in Part (i), but the received signal is

$$Y(t) = \frac{3}{4} \mathsf{A} X\, s(t) + N(t), \quad t \in \mathbb{R}.$$

(You were misinformed about the amplitude of the signal.) What is the probability of error now?