



Exercise 8 of November 9, 2016

<http://www.isi.ee.ethz.ch/teaching/courses/it1/>

Problem 1

Data Processing

Let $X_1 \text{---} X_2 \text{---} X_3 \text{---} \dots \text{---} X_n$ form a Markov chain, i.e.,

$$P_{X_1 X_2 \dots X_n}(x_1, x_2, \dots, x_n) = P_{X_1}(x_1) P_{X_2|X_1}(x_2|x_1) \cdots P_{X_n|X_{n-1}}(x_n|x_{n-1}).$$

Reduce $I(X_1; X_2, \dots, X_n)$ to its simplest form.

Problem 2

Preprocessing the Output

A communication channel with transition probabilities $W(\cdot|\cdot)$ and channel capacity

$$C = \max_{P_X} I(X; Y)$$

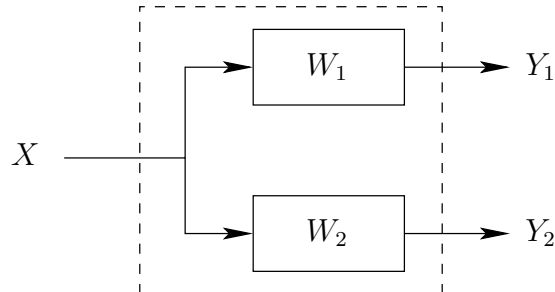
is given. A helpful statistician preprocesses the output by forming $\tilde{Y} = g(Y)$. He claims that this will strictly improve the capacity.

- Show that he is wrong.
- Under what conditions does he not strictly decrease the capacity?

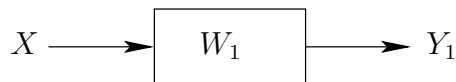
Problem 3***A Channel With Two Independent Looks at Y***

Let Y_1 and Y_2 be conditionally independent given X .

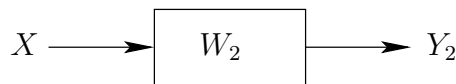
- Show that $I(X; Y_1, Y_2) = I(X; Y_1) + I(X; Y_2) - I(Y_1; Y_2)$.
- Conclude that the capacity of the channel



is upper bounded by the sum of the capacity of the channel



and the capacity of the channel

**Problem 4*****Miscellaneous Capacities***

Find the capacity and an optimizing input probability assignment for each of the discrete memoryless channels in Figure 1.

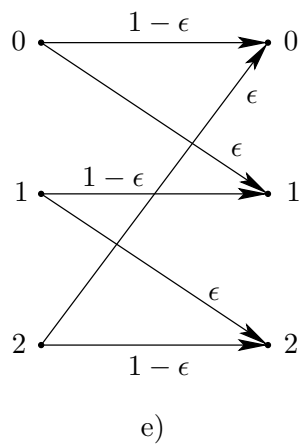
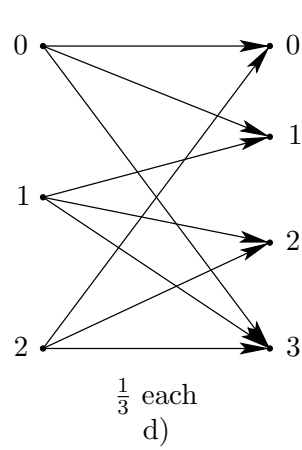
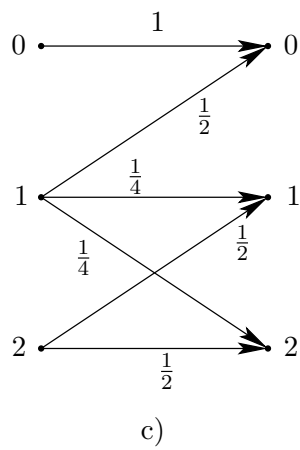
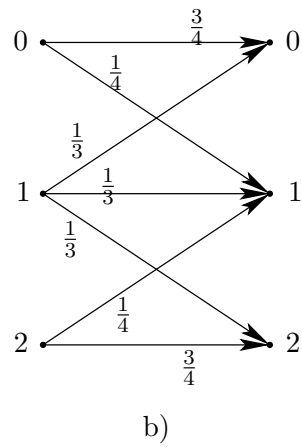
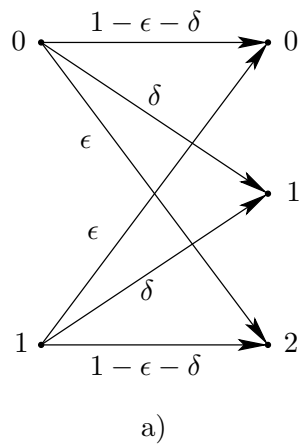


Figure 1: Miscellaneous channels.