Information Theory I

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http://www.isi.ee.ethz.ch/teaching/courses/it1/

Problem 1 — *Source Coding for Bernoulli Sources*

Let $X_1, \ldots, X_n$ be IID $\sim$ Bernoulli(p) and let $Z_1, \ldots, Z_n$ be IID $\sim$ Bernoulli(r) independent of $X_1, \ldots, X_n$. Let $Y_i = X_i \oplus Z_i$ (bitwise mod 2 addition) for $i = 1, \ldots, n$. Suppose Encoder 1 describes $X_1, \ldots, X_n$ at rate R_1 and Encoder 2 describes $Y_1, \ldots, Y_n$ at rate R_2. What region of rate pairs $(\mathsf{R}_1, \mathsf{R}_2)$ allows recovery of $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_n$ with probability of error tending to zero as n tends to infinity?

Problem 2 — *Slepian-Wolf*

Let (X, Y) have the joint probability mass function $p(x, y)$ given in Table 1 where $\beta = \frac{1}{6} - \frac{\alpha}{2}$ and where $0 \leq \alpha \leq \frac{1}{3}$ is some given parameter.

Table 1: Joint probability mass function $p(x, y)$.

		X		
		1	2	3
	1	α	β	β
Y	2	β	α	β
	3	β	β	α

a) Find the Slepian-Wolf rate region for this source and draw it graphically.

b) What is $\Pr[X = Y]$ in terms of α?

c) What is the rate region if $\alpha = \frac{1}{3}$?

d) What is the rate region if $\alpha = \frac{1}{9}$?

Problem 3 — *Slepian-Wolf*

Two senders know random variables U_1 and U_2, respectively. Let the random variables (U_1, U_2) have the joint distribution shown in Table 2. Here $\alpha + \beta + \gamma = 1$. Find the region of rate pairs $(\mathsf{R}^{(1)}, \mathsf{R}^{(2)})$ that would allow a common receiver to decode both random variables reliably.

Table 2: Joint probability mass function $P_{U_1,U_2}(\cdot,\cdot)$.

		U_2				
		1	2	3	$\cdots$	m
	1	α	$\frac{\beta}{m-1}$	$\frac{\beta}{m-1}$	$\cdots$	$\frac{\beta}{m-1}$
	2	$\frac{\gamma}{m-1}$	0	0	$\cdots$	0
U_1	3	$\frac{\gamma}{m-1}$	0	0	$\cdots$	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
	m	$\frac{\gamma}{m-1}$	0	0	$\cdots$	0