Information Theory I

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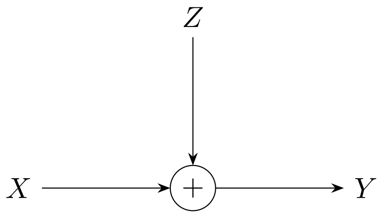
http://www.isi.ee.ethz.ch/teaching/courses/it1.html

Problem 1 — ***An Additive Noise Channel***

Find the channel capacity of the following discrete memoryless channel:



where $\Pr[Z = 0] = \Pr[Z = a] = \frac{1}{2}$ for some fixed value $a \in \mathbb{R}$. The input X takes values in the binary alphabet $\mathcal{X} = \{0, 1\}$. Assume that Z is independent of X.
Hint: Observe that the channel capacity depends on the value of a, i.e., you need to introduce a case distinction!

Problem 2 — ***Data Processing***

Let $X_1 \multimap X_2 \multimap X_3 \multimap \cdots \multimap X_n$ form a Markov chain, i.e.,

$$P_{X_1 X_2 \ldots X_n}(x_1, x_2, \ldots, x_n) = P_{X_1}(x_1) P_{X_2|X_1}(x_2|x_1) \cdots P_{X_n|X_{n-1}}(x_n|x_{n-1}).$$

Reduce $I(X_1; X_2, \ldots, X_n)$ to its simplest form.

Problem 3 — ***Preprocessing the Output***

A communication channel with transition probabilities $W(\cdot|\cdot)$ and channel capacity

$$\mathsf{C} = \max_{P_X} I(X;Y)$$

is given. A helpful statistician preprocesses the output by forming $\tilde{Y} = g(Y)$. He claims that this will strictly improve the capacity.

a) Show that he is wrong.

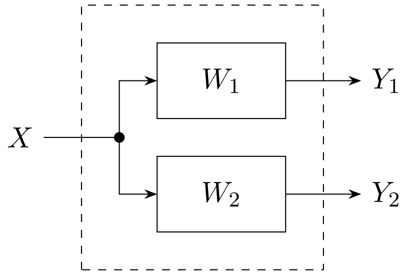
b) Under what conditions does he not strictly decrease the capacity?

Problem 4 — *A Channel With Two Independent Looks at Y*

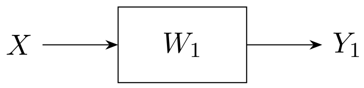
Let Y_1 and Y_2 be conditionally independent given X.

a) Show that $I(X; Y_1, Y_2) = I(X; Y_1) + I(X; Y_2) - I(Y_1; Y_2)$.

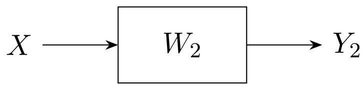
b) Conclude that the capacity of the channel



is upper bounded by the sum of the capacity of the channel



and the capacity of the channel



Problem 5 — *Miscellaneous Capacities*

Find the capacity and an optimizing input probability assignment for each of the discrete memoryless channels in Figure 1.

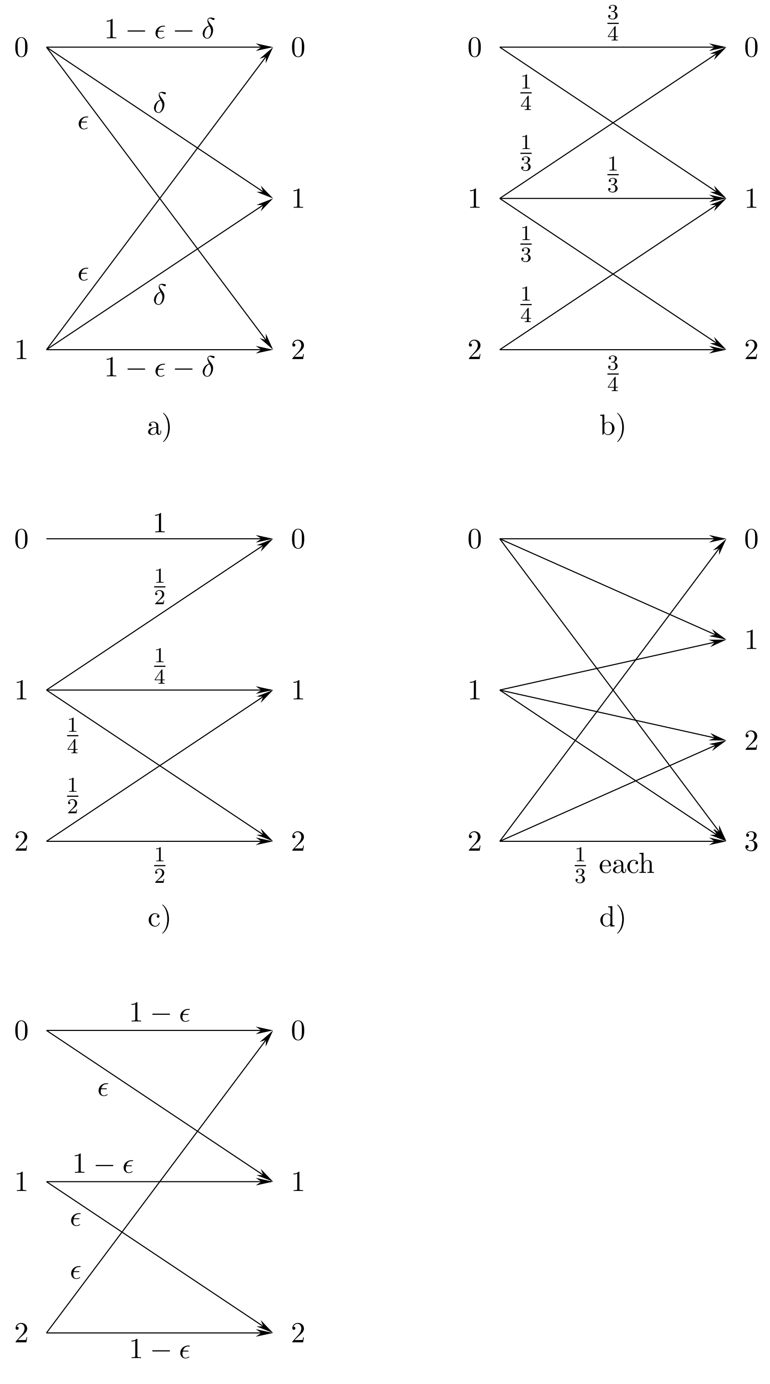


Figure 1: Miscellaneous channels.