



Exercise 11 of November 29, 2017

<http://www.isi.ee.ethz.ch/teaching/courses/it1.html>

Problem 1

Properties of $R(D)$

Consider an IID source with distribution P_X and a distortion measure $d(x, \hat{x})$. Let $R(D)$ be the rate distortion function for this setup. Define a new distortion measure $d'(x, \hat{x}) \triangleq d(x, \hat{x}) - w(x)$ where $w: \mathcal{X} \rightarrow \mathbb{R}$ is such that $d'(x, \hat{x}) \geq 0$ for all x and \hat{x} . Let $R'(D)$ be the rate distortion function corresponding to P_X and d' .

a) Show that

$$R'(D) = R(D + \bar{w}),$$

$$\text{where } \bar{w} \triangleq \sum_{x \in \mathcal{X}} P_X(x)w(x).$$

b) Show that for every distortion measure d , there exists a function w such that

$$\min_{\hat{x} \in \hat{\mathcal{X}}} d'(x, \hat{x}) = 0$$

holds for every $x \in \mathcal{X}$.

c) Let $P_X(0) = P_X(1) = \frac{1}{2}$, and let d be defined as

$d(x, \hat{x})$	$\hat{x} = 0$	$\hat{x} = 1$
$x = 0$	1	2
$x = 1$	3	4

Find $R(D)$.

Problem 2

Erasure Distortion

Consider $X \sim \text{Bernoulli}(\frac{1}{2})$. Let \hat{X} take values in $\hat{\mathcal{X}} = \{0, 1, ?\}$, and let the distortion measure be given by

$$d(x, \hat{x}) \triangleq \begin{cases} 0 & \text{if } x = \hat{x}, \\ 1 & \text{if } \hat{x} = ?, x \in \{0, 1\}, \\ \infty & \text{if } \hat{x} = 0, x = 1 \text{ or } \hat{x} = 1, x = 0, \end{cases}$$

so the distortion is 1 for an “erasure” and ∞ for an “error”.

a) Calculate the rate distortion function for this source and plot the rate distortion region.

b) Can you suggest a simple scheme to achieve any value of the rate distortion function for this source?

Problem 3***Rate Distortion Function with Infinite Distortion***

Find the rate distortion function for $X \sim \text{Bernoulli}(1/2)$ and the distortion measure

$$d(x, \hat{x}) = \begin{cases} 0 & \text{if } x = \hat{x}, \\ 1 & \text{if } x = 1 \text{ and } \hat{x} = 0, \\ \infty & \text{if } x = 0 \text{ and } \hat{x} = 1. \end{cases}$$

Problem 4***Rate Distortion for Uniform Source with Hamming Distortion***

Consider a source X that is uniformly distributed over $\{1, 2, \dots, m\}$ and the Hamming distortion

$$d(x, \hat{x}) = \begin{cases} 0 & \text{if } x = \hat{x}, \\ 1 & \text{if } x \neq \hat{x}. \end{cases}$$

Find the rate distortion function.