Information Theory I

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Prof. Dr. A. Lapidoth

Signal and Information Processing Laboratory
Institut für Signal- und Informationsverarbeitung

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http://www.isi.ee.ethz.ch/teaching/courses/it1.html

Problem 1 — *Source Coding for Bernoulli Sources*

Let $X_1, \ldots, X_n$ be IID $\sim$ Bernoulli(p), and let $Z_1, \ldots, Z_n$ be IID $\sim$ Bernoulli(r) independent of $X_1, \ldots, X_n$. For all $i \in \{1, \ldots, n\}$, define $Y_i \triangleq X_i \oplus Z_i$. Suppose one encoder describes $X_1, \ldots, X_n$ at rate R_X and the other encoder describes $Y_1, \ldots, Y_n$ at rate R_Y. What region of rate pairs (R_X, R_Y) allows recovery of $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_n$ with probability of error tending to zero as n tends to infinity?

Problem 2 — *Slepian–Wolf*

Let (X, Y) have the joint probability mass function $p(x, y)$ given in Table 1 where $\beta = \frac{1}{6} - \frac{\alpha}{2}$ and where $0 \leq \alpha \leq \frac{1}{3}$ is some given parameter.

Table 1: Joint probability mass function $p(x, y)$.

		X = 1	X = 2	X = 3
Y	1	α	β	β
	2	β	α	β
	3	β	β	α

a) Find the Slepian–Wolf rate region for this source and draw it graphically.

b) What is $\Pr[X = Y]$ in terms of α?

c) What is the rate region if $\alpha = \frac{1}{3}$?

d) What is the rate region if $\alpha = \frac{1}{9}$?

Problem 3 *Slepian–Wolf*

Determine the Slepian–Wolf rate region for the joint probability mass function shown in Table 2 where $\alpha \geq 0$, $\beta \geq 0$, and $\gamma \geq 0$ are such that $\alpha + \beta + \gamma = 1$.

Table 2: Joint probability mass function P_{XY}.

		Y				
		1	2	3	$\cdots$	m
	1	α	$\frac{\beta}{m-1}$	$\frac{\beta}{m-1}$	$\cdots$	$\frac{\beta}{m-1}$
	2	$\frac{\gamma}{m-1}$	0	0	$\cdots$	0
X	3	$\frac{\gamma}{m-1}$	0	0	$\cdots$	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
	m	$\frac{\gamma}{m-1}$	0	0	$\cdots$	0