



Selected Questions

<http://www.isi.ee.ethz.ch/teaching/courses/it1>

Problem 1

Label all of the following statements with \leq or \geq such that the resulting inequality is valid. When does equality hold in each inequality?

$$H(X, Y|Z) \quad ? \quad H(X|Z) \quad (1)$$

$$I(X, Y; Z) \quad ? \quad I(X; Z) \quad (2)$$

$$H(X, Y, Z) - H(X, Y) \quad ? \quad H(X, Z) - H(X) \quad (3)$$

Problem 2

Consider a commander of an army besieged at a fort for whom the only means of communication to his allies is a set of carrier pigeons. Assume that each carrier pigeon can carry one letter (8 bits), and assume that a pigeon is released once every 5 minutes, and that it takes exactly 3 minutes to reach its destination.

1. Assuming all the pigeons reach safely, what is the capacity of this link in bits/hour?
2. Now assume that the enemies try to shoot down the pigeons, and that they manage to hit a fraction α of them. Since the pigeons are sent at a constant rate, the receiver knows when the pigeons are missing. What is the capacity of this link?
3. Now assume that the enemy is even more cunning, and every time they shoot down a pigeon, they send out a dummy pigeon carrying a random letter (chosen uniformly from all 8-bit letters). What is the capacity of this link in bits/hour?

Set up an appropriate model for the channel in each of the above cases, and indicate how to go about finding the capacity.

Problem 3

Suppose that $\mathcal{X} = \hat{\mathcal{X}} = \{1, 2, 3, 4\}$. Consider an IID source emitting symbols uniformly over \mathcal{X} . The distortion measure $d(x, \hat{x})$ is given by

$d(x, \hat{x})$	$\hat{x} = 1$	$\hat{x} = 2$	$\hat{x} = 3$	$\hat{x} = 4$
$x = 1$	0	0	1	1
$x = 2$	0	0	1	1
$x = 3$	1	1	0	0
$x = 4$	1	1	0	0

Find $R(D)$.