Problem 1

Find the capacity region for each of the following multiple-access channels:

a) Additive modulo-2 MAC: \( X^{(1)}, X^{(2)} \in \{0, 1\} \) and \( Y = X^{(1)} \oplus X^{(2)} \).

b) Multiplicative MAC: \( X^{(1)}, X^{(2)} \in \{-1, 1\} \) and \( Y = X^{(1)} \cdot X^{(2)} \).

Problem 2

Consider the following multiple-access channel with \( X^{(1)} = X^{(2)} = Y = \{0, 1\} \): If \( (X^{(1)}, X^{(2)}) = (0, 0) \), then \( Y = 0 \). If \( (X^{(1)}, X^{(2)}) = (0, 1) \), then \( Y = 1 \). If \( (X^{(1)}, X^{(2)}) = (1, 0) \), then \( Y = 1 \). If \( (X^{(1)}, X^{(2)}) = (1, 1) \), then \( Y = 0 \) with probability \( \frac{1}{2} \) and \( Y = 1 \) with probability \( \frac{1}{2} \).

a) Show that the rate pairs \((1, 0)\) bits and \((0, 1)\) bits are achievable.

b) Show that for any nondegenerate distribution \( Q_{X^{(1)}} \cdot Q_{X^{(2)}} \) we have

\[ I(X^{(1)}, X^{(2)}; Y) < 1 \text{ bit} \]

for any product distribution \( Q_{X^{(1)}} \cdot Q_{X^{(2)}} \). Hence the operation of convexification strictly enlarges the capacity region. This channel was introduced independently by Csiszár/Körner and Bierbaum/Wallmeier.

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Problem 3

Cooperative Capacity of a MAC

a) Suppose that both senders have access to both indices $W^{(1)} \in \{1, \ldots, e^{nR_1}\}$ and $W^{(2)} \in \{1, \ldots, e^{nR_2}\}$. Thus the codewords $X^{(1)}(W^{(1)}, W^{(2)})$ and $X^{(2)}(W^{(1)}, W^{(2)})$ depend on both indices. Find the capacity region.

b) Evaluate this region for the binary erasure multiple access channel

$$Y = X^{(1)} + X^{(2)},$$

where $X^{(i)} \in \{0, 1\}$ and where the addition is standard addition in $\mathbb{N}_0$. Compare to the noncooperative region.