Information Theory II

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http://www.isi.ee.ethz.ch/teaching/courses/it2.html

Problem 1 — *"Double"-Gaussian Channel*

a) Consider a discrete-time memoryless Gaussian channel with average-power constraint $\mathsf{E_s} = 1$. Sketch the capacity of this channel as a function of the noise variance N.

b) You are given two parallel independent Gaussian channels with noise variances N_1 and N_2, respectively. You are allocated a total power of $\mathsf{E_s}$ to use between these two channels. The sum of the noise variances in the two channels is $2\mathsf{N}$, but you will be able to choose from the two schemes below as to how the noise is distributed (Δ and N are fixed).

Scheme A: $\mathsf{N}_1 = \mathsf{N}_2 = \mathsf{N}$,
Scheme B: $\mathsf{N}_1 = \mathsf{N} + \Delta, \quad \mathsf{N}_2 = \mathsf{N} - \Delta, \quad 0 < \Delta < \mathsf{N}$.

i) If you must put half of your power allocation in each channel, which scheme would you prefer?

ii) If you are allowed to arbitrarily divide your power allocation between the two channels, which scheme would you prefer?

Problem 2 — *Bandlimited Gaussian Channel*

Consider the bandlimited Gaussian channel with noise power spectral density $\mathsf{N}_0/2$ and power P. The capacity of this channel is

$$W \log\left(1 + \frac{\mathsf{P}}{\mathsf{N}_0 W}\right) \quad \text{bits per second,}$$

where W is the bandwidth of the channel. What would you rather have, twice the bandwidth or twice the power?

Problem 3 — *Exponential Noise Channel*

Consider the channel

$$Y_k = X_k + Z_k,$$

where the inputs $\{X_k\}$ are nonnegative and where $\{Z_k\}$ are IID exponentially distributed with mean μ. Assume that the inputs satisfy the mean constraint

$$\mathsf{E}[X_k] \leq \lambda, \quad k = 0, 1, \ldots$$

Show that the capacity of this channel is

$$\mathsf{C} = \log\left(1 + \frac{\lambda}{\mu}\right).$$