Information Theory II

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http://www.isi.ee.ethz.ch/teaching/courses/it2.html

Problem 1 — *Cooperation on the Gaussian MAC*

Derive the capacity region of the Gaussian MAC with cooperation. More specifically, derive the capacity region of the MAC

$$Y = X_1 + X_2 + Z,$$

where both transmitters have access to both messages, $Z \sim \mathcal{N}(0, \sigma^2)$, and for every pair of messages, the average power of X_1 must not exceed E_1 and the average power of X_2 must not exceed E_2.

Problem 2 — *Gaussian MAC*

Consider a Gaussian MAC

$$Y = X_1 + X_2 + Z,$$

where $X_1 \in \mathbb{R}$ is the input from transmitter 1, $X_2 \in \mathbb{R}$ is the input from transmitter 2, both transmitters are independent, Z is a Gaussian random variable with mean zero and variance σ^2, and Y is the channel output. We impose average-power constraints on both inputs, i.e., the average power of every codeword from transmitter 1 must not exceed E_1 and the average power of every codeword from transmitter 2 must not exceed E_2. Show that the capacity region of this channel is the set of all rate pairs $(\mathsf{R}_1, \mathsf{R}_2)$ satisfying

$$\begin{aligned} \mathsf{R}_1 &\leq \frac{1}{2}\log\left(1 + \frac{\mathsf{E}_1}{\sigma^2}\right), \\ \mathsf{R}_2 &\leq \frac{1}{2}\log\left(1 + \frac{\mathsf{E}_2}{\sigma^2}\right), \\ \mathsf{R}_1 + \mathsf{R}_2 &\leq \frac{1}{2}\log\left(1 + \frac{\mathsf{E}_1 + \mathsf{E}_2}{\sigma^2}\right). \end{aligned}$$

Problem 3 — *Gaussian MAC and TDMA*

Suppose that on a Gaussian MAC with equal average-power constraints $\mathsf{E}_1 = \mathsf{E}_2 = \mathsf{E}$ a rate pair (R, R) is in the interior of the capacity region. Show that this rate pair can be achieved by *time-division multiplexing*, where at each channel use, at most one input of the MAC is nonzero.