Problem 1

Asynchronous Multiple-Access Channel Capacity

a) The main assumption is that there exists a bound on the maximum delay of each user (which is then known to the receiver). For practical systems this assumption is reasonable since usually one may assume that two different clocks may agree on the current date, for example, even if they disagree on the current seconds and minutes. On the other hand, considering the fact that the system depends on the assumption that the block length is much longer than the maximum delay, this assumption results in a required block length of several hours or even days, which is completely unrealistic.

b) The parameter $\alpha$ serves as time-sharing parameter. It is necessary in order to be able to achieve the convex capacity region of the MAC, which otherwise might not be achievable!

c) The main idea of the proof is that since the maximum delay is bounded, there is a maximum overlap of different frames of the two users at the receiver. If now the blocklength is much larger than the maximum delay, this overlap is negligible and is therefore ignored at the receiver.

d) The authors then let the maximum delay tend to infinity while still maintaining the assumption that the blocklength is much larger than the maximum delay, i.e., they assume a sequence of delays $d_j$ such that

$$\lim_{j \to \infty} d_j = \infty,$$

$$\lim_{j \to \infty} \frac{d_j}{n_j} = 0.$$  

(1) 

(2)

e) The channel is not totally asynchronous because of (2) which shows that $D$ will never take on too large values in comparison to the blocklength $n$.

f) Actually, this is the main drawback of all results touched here: In practice it is much harder to keep the symbols synchronized than the frames simply because the symbol duration is much shorter than the frame duration. Hence, one should investigate the continuous-time channel and consider the situation when the symbols are not in phase!

g) The capacity region of the totally asynchronous MAC corresponds to the standard MAC capacity region (in the form of our first theorem in class), however, without the convex hull operation.