On the Capacity of Non-Coherent Fading Networks

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Abstract — **We obtain the first term in the high signal-to-noise ratio (SNR) expansion of the capacity of fading networks where the transmitters and receivers—while fully cognizant of the fading *law*—have no access to the fading *realization*. This term is an integer multiple of $\log\log \mathrm{SNR}$ with the coefficient having a simple combinatorial characterization.**

I. The Channel Model

The communication problem we address is one with $n_\mathrm{T} \in \mathbb{N}$ transmitters ($\mathbb{N}$ denoting the positive integers) and $n_\mathrm{R} \in \mathbb{N}$ receivers. Denoting by $X_k^{(t)} \in \mathbb{C}$ the complex signal transmitted at time-k by Transmitter t and by $Y_k^{(r)} \in \mathbb{C}$ the time-k received signal by Receiver r, we assume a fading channel model

$$\mathbf{Y}_k = \mathbb{H}_k \mathbf{X}_k + \mathbf{Z}_k \tag{1}$$

where $\mathbf{Y}_k$ and $\mathbf{X}_k$ denote the time-k vectors of received and transmitted signals $(Y_k^{(1)}, \ldots, Y_k^{(n_\mathrm{R})})^\mathsf{T}$, $(X_k^{(1)}, \ldots, X_k^{(n_\mathrm{T})})^\mathsf{T}$, respectively, $\mathbb{H}_k$ is a random $n_\mathrm{R} \times n_\mathrm{T}$ complex matrix, and $\{\mathbf{Z}_k\}$ are IID n_R-dimensional complex Gaussian vectors of IID variance-1 circularly symmetric complex Gaussian components.

So far, the model is very much reminiscent of the multiple antenna fading channel. The difference here is that we assume that some of the components of the fading matrix are deterministically zero. Such entries model situations where some receivers only receive the signals transmitted by a subset of the transmitters. We denote by

$$\mathcal{Z} \subseteq \{1, \cdots, n_\mathrm{R}\} \times \{1, \cdots, n_\mathrm{T}\}$$

the set of components of the matrix-valued fading process $\{\mathbb{H}_k\}$ that are deterministically zero:

$$H_k^{(r,t)} = 0 \quad \text{a.s. whenever } (r,t) \in \mathcal{Z} \tag{2}$$

and we denote by

$$\mathbf{H}_k^{\mathcal{Z}} = \{H_k^{(r,t)} : (r,t) \notin \mathcal{Z}\}$$

the rest of the components.

We shall assume that the process $\{\mathbf{H}_k^{\mathcal{Z}}\}$ is a stationary and ergodic Gaussian process of finite second moment

$$\mathsf{E}\left[\left|H_1^{(r,t)}\right|^2\right] < \infty, \quad 1 \le r \le n_\mathrm{R}, 1 \le t \le n_\mathrm{T} \tag{3}$$

and of finite differential entropy rate

$$h\left(\{\mathbf{H}_k^{\mathcal{Z}}\}\right) > -\infty. \tag{4}$$

II. The Main Result

Denote by $C_\mathrm{co}(\mathcal{E}_\mathrm{s})$ the capacity of the channel from all the transmitters to all the receivers when the average transmitted power by each transmitter is limited to $\mathcal{E}_\mathrm{s}$. Denote by $C_\mathrm{MAC}(\mathcal{E}_\mathrm{s}) \le C_\mathrm{co}(\mathcal{E}_\mathrm{s})$ the sum-rate capacity of this channel when the different transmitters are viewed as separate users, each of average power $\mathcal{E}_\mathrm{s}$, in a multiple-access channel whose output is the vector of all received signals.

Our main results are that under the above assumptions on $\{\mathbb{H}_k\}$

$$\varlimsup_{\mathcal{E}_\mathrm{s} \to \infty} \{C_\mathrm{co}(\mathcal{E}_\mathrm{s}) - \kappa^* \log\log \mathcal{E}_\mathrm{s}\} < \infty \tag{5}$$

and

$$\varlimsup_{\mathcal{E}_\mathrm{s} \to \infty} \{\kappa^* \log\log \mathcal{E}_\mathrm{s} - C_\mathrm{MAC}(\mathcal{E}_\mathrm{s})\} < \infty \tag{6}$$

where the non-negative integer κ^* can be computed in the following combinatorial way. For any transmitter $1 \le t \le n_\mathrm{T}$ let $\mathcal{R}_t$ denote the set of receivers its signal affects

$$\mathcal{R}_t = \{1 \le r \le n_\mathrm{R} : (r,t) \notin \mathcal{Z}\}, \quad 1 \le t \le n_\mathrm{T}.$$

Define a κ-length power chain

$$(t_1, \ldots, t_\kappa) \in \{1, \ldots, n_\mathrm{T}\}^\kappa$$

as a κ-tuple satisfying $\mathcal{R}_{t_1} \neq \emptyset$, $\mathcal{R}_{t_2} \setminus \mathcal{R}_{t_1} \neq \emptyset$, $\mathcal{R}_{t_3} \setminus (\mathcal{R}_{t_1} \cup \mathcal{R}_{t_2}) \neq \emptyset$, and in general

$$\mathcal{R}_{t_\nu} \setminus \bigcup_{1 \le \eta < \nu} \mathcal{R}_{t_\eta} \neq \emptyset, \quad \nu = 1, \ldots, \kappa. \tag{7}$$

Then κ^* is the length of the longest power chain.

Note that in the special case that $\{\mathbb{H}_k\}$ is of finite differential entropy rate (and hence has no deterministically zero components) the longest power chain is of length one, and our results are in agreement with the results on multiple-antenna fading channels [1]. The present results allow us, however, to analyze much more interesting networks such as Wyner's cellular communication model [2], [3], etc.

References

[1] A. Lapidoth and S. Moser, "Capacity bounds via duality with applications to multi-antenna systems on flat fading channels," *IEEE Trans. on Inform. Theory*, pp. 2426–2467, Oct. 2003.

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