Limits on Reliable Communication over Flat-Fading Channels

Amos Lapidoth Stefan M. Moser
Institute for Information and Signal Processing
Swiss Federal Institute of of Technology
CH-8092 Zurich, Switzerland
{lapidoth,moser}@isi.ee.ethz.ch

Abstract — **Upper bounds on the capacity of flat fading channels are derived. The results apply to Rayleigh fading, to Ricean fading, and to more general circularly symmetric ergodic Gaussian fading. It is demonstrated that, contrary to the additive Gaussian noise channel where capacity grows logarithmically in the signal-to-noise ratio (SNR), the capacity of these channels typically grows only double-logarithmically in the SNR. Multi-antenna communication is also discussed, as also non-Gaussian fading.**

I. Introduction

This paper studies the capacity of pass-band pulse-amplitude modulation over a "flat-fading" channel, i.e., a channel of "delay spread" far smaller than the pulse duration. The tools we employ are, however, more general, and it is hoped that they may be of use in the capacity calculation of other channels as well.

The channel we study is a discrete-time channel whose complex-valued output Y_k at time k is given by

$$Y_k = dx_k + A_k x_k + Z_k, \tag{1}$$

where x_k is the complex-valued channel input at time k; the constant d is a deterministic complex number; the sequence $\{A_k\}$ is a zero-mean stationary ergodic circularly-symmetric complex-Gaussian stochastic process; and the sequence $\{Z_k\}$ is a sequence of independent and identically distributed (IID) complex-Gaussian random variables of zero mean and variance $\mathcal{N}$. It is assumed throughout that the fading process $\{A_k\}$ is independent of the additive-noise process $\{Z_k\}$, and that their joint law does not depend on the input sequence $\{x_k\}$. The inputs are assumed to be average-power limited to $\mathcal{P}$, but the proposed technique allows for the bounds to be further tightened to account for additional peak-power constraints.

It should be noted that we only allow for the transmitted signal $\mathbf{x}(w)$ to depend on the message w, and not on the realization of the fading sequence $\{A_k\}$. Similarly, the decoder's output must be a function of the received sequence $\{Y_k\}$ only, and may not depend on the realization of the fading sequence. In other words, the sequence $\{A_k\}$ is assumed to be unknown to both transmitter and receiver. This assumption is often referred to in the literatures as the "no side-information" assumption.

We normalize the problem by assuming that $d \geq 0$ is real, and that

$$\mathsf{E}\left[|A_k|^2\right] = 1.$$

The model under consideration is fairly general and includes, as special cases:

- Rayleigh Fading where $\{A_k\}$ are IID and $d = 0$ [1–3];
- Ricean Fading where $\{A_k\}$ are still IID but $d > 0$.

II. The Main Results

For Rayleigh fading channels with m receive antennas we obtain the bound

$$C_{\mathrm{u}} = \inf_{\alpha>0}\Big\{ m\big(\psi(m) - 1\big) - \log\big((m-1)!\big) + \alpha\Big(\log\Big(m\big(1+\mathcal{P}/\mathcal{N}\big)\Big) + 1 - \psi(m)\Big) - \alpha\log\alpha + \log\Gamma(\alpha)\Big\},$$

where

$$\psi(k) = \frac{\Gamma'(k)}{\Gamma(k)} = -\gamma + \sum_{j=1}^{k-1}\frac{1}{j},$$

which yields a double-logarithmic growth of the capacity in the SNR. This behavior does not rely heavily on the Rayleigh assumption; it holds, for example, in the single-antenna case whenever the fading process is a zero-mean circularly symmetric IID fading process with a random fading magnitude that has a bounded density.

By computing firm upper bounds on the capacity of the Ricean channel, we also demonstrate that the double-logarithmic behavior continues to hold even in the presence of a specular component $d > 0$. Finally, we tackle the case where the fading process has memory and obtain upper bounds that demonstrate:

Proposition 1. *Assume the channel model of (1) where the sequence $\{A_k\}$ is a stationary and ergodic circularly symmetric Gaussian random process satisfying*

$$\lim_{n\to\infty}\mathsf{E}\left[\Big|A_n - \mathsf{E}[A_n|A_1,\ldots,A_{n-1}]\Big|^2\right] > 0.$$

Then,

$$\lim_{\mathcal{P}\uparrow\infty}\ \limsup_{n\to\infty}\ \sup\ \frac{n^{-1}I(\mathbf{X};\mathbf{Y})}{\log\Big(1+\log\big(1+\frac{\mathcal{P}_{\mathrm{out}}}{\mathcal{N}}\big)\Big)} \leq 1,$$

where the supremum is over all input distributions on $X_1,\ldots,X_n$ satisfying the average power constraint

$$\mathsf{E}\Big[\mathbf{X}^\dagger\mathbf{X}\Big] \leq n\mathcal{P},$$

and where

$$\mathcal{P}_{\mathrm{out}} = \mathcal{P}(d^2+1).$$

References

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