On the Ricean Fading Multi-Access Channel

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Abstract — **A two-user Ricean fading multi-access channel is considered without state-information at the receiver or the transmitter. The sum-rate fading number is defined and it is shown that if neither specular component is zero, then it is strictly smaller than the fading number that would result if the users could fully cooperate. If one of the users is required to use circularly-symmetric signaling, then the fading number is achievable by time-sharing. The problem of computing the sum-rate fading number without this constraint is still open.**

I. Channel Model

We consider a memoryless Ricean two-user multi-access channel (MAC) of complex inputs x_1 and x_2 and complex output Y

$$Y = (d_1 + H_1)x_1 + (d_2 + H_2)x_2 + Z. \tag{1}$$

Here, $H_i \sim \mathcal{N}_{\mathbb{C}}(0,1)$, $i = 1, 2$, and $Z \sim \mathcal{N}_{\mathbb{C}}\big(0, \sigma^2\big)$ are independent complex circularly symmetric Gaussian random variables and d_1, d_2 are deterministic. We assume that while the probability laws of H_1, H_2, and Z are known to both transmitters and receiver, their realization is unknown. The signals transmitted by the two users are assumed to be average power limited. We shall be mostly interested in the symmetric case where the average power constraints for the two users are identical, i.e.,

$$\mathsf{E}\big[|X_i|^2\big] \leq \mathcal{E}_{\mathrm{s}}, \quad i = 1, 2. \tag{2}$$

Since rotating the inputs signals does not affect their average power, we shall assume without loss of generality $d_1, d_2 \geq 0$.

We define the sum-rate capacity as

$$C_{\mathrm{Tot}}(\mathcal{E}_{\mathrm{s}}) = \sup I(X_1, X_2; Y) \tag{3}$$

where the supremum is over all pairs of independent random variables X_1, X_2 satisfying (2). We define the sum-rate fading number χ_{MAC} as:

$$\chi_{\mathrm{MAC}} = \varlimsup_{\mathcal{E}_{\mathrm{s}} \to \infty} \left\{ C_{\mathrm{Tot}}(\mathcal{E}_{\mathrm{s}}) - \log\log\frac{\mathcal{E}_{\mathrm{s}}}{\sigma^2} \right\}. \tag{4}$$

II. Some Bounds on χ_{MAC}

If X_2 is deterministically zero, then the channel $x_1 \mapsto Y$ is a single-user Ricean channel with corresponding fading number [1] $\log(d_1^2) - \mathrm{Ei}(-d_1^2) - 1$, where

$$\mathrm{Ei}(-z) = -\int_z^\infty \frac{e^{-t}}{t}\,\mathrm{d}t, \qquad z > 0. \tag{5}$$

An analogous result can be obtained by setting $X_1 = 0$. This approach yields the lower bound

$$\chi_{\mathrm{MAC}} \geq \log(d_\ell^2) - \mathrm{Ei}(-d_\ell^2) - 1, \tag{6}$$

where $d_\ell^2 = \max\{d_1^2, d_2^2\}$.

An upper bound can be derived by considering the case where the users can cooperate. This situation is reminiscent of a single-user channel with two transmit antennae and one receive antenna, which was analyzed in [1], and yields

$$\chi_{\mathrm{MAC}} \leq \log(d_{\mathrm{u}}^2) - \mathrm{Ei}(-d_{\mathrm{u}}^2) - 1, \tag{7}$$

where $d_{\mathrm{u}}^2 = d_1^2 + d_2^2$.

The upper and lower bound agree if, and only if, $d_1 \cdot d_2 = 0$. We shall henceforth focus on the more difficult case $d_1 \cdot d_2 \neq 0$.

III. Additional Results

For the general case where $d_1 \cdot d_2 \neq 0$ we were, as of yet, unable to compute the rate-sum fading number χ_{MAC}. We present here two partial results. The first states that for $d_1 \cdot d_2 \neq 0$ the full-cooperation bound (7) is strict:

$$\chi_{\mathrm{MAC}} < \log(d_{\mathrm{u}}^2) - \mathrm{Ei}(-d_{\mathrm{u}}^2) - 1, \tag{8}$$

where $d_{\mathrm{u}}^2 = d_1^2 + d_2^2$.

The second results deals with circularly-symmetric inputs. Let $\chi_{\mathrm{MAC}}^{\text{c-s}}$ be defined as in (4) with $C_{\mathrm{Tot}}(\mathcal{E}_{\mathrm{s}})$ replaced by a supremum similar to (3) but with additional constraint that

$$\mathsf{E}\big[X_1 \bigm| |X_1|\big] = 0. \tag{9}$$

Then $\chi_{\mathrm{MAC}}^{\text{c-s}}$ is identical to the right hand side of (6). That is, the maximum throughput is achieved by requiring that one of the users be silent.

IV. Conclusions

While there are some indications that the lower bound (6) is tight, we were unable to prove this except for the case $d_1 \cdot d_2 = 0$ or under the additional constraint (9). The difficulty seems to be that (3) is a non-concave problem.

References

[1] Amos Lapidoth and Stefan M. Moser. Capacity bounds via duality with applications to multi-antenna systems on flat fading channels. Submitted to *IEEE Transactions on Information Theory,* available at <http://www.isi.ee.ethz.ch/~moser>, June 25, 2002.