Gaussian Broadcast Channel with Partial Feedback

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***Abstract*—We present new achievable regions for the two-user Gaussian broadcast channel with noiseless feedback from one of the users. Our regions improve on previous achievable regions.**

Classification: Information theory, communications.

I. Channel Model

We consider a two-receiver broadcast scenario where a single transmitter wishes to send Message M_1 to Receiver 1 and an independent message M_2 to Receiver 2. The messages M_1 and M_2 are assumed to be uniformly distributed over the sets $\{1, \ldots, \lfloor 2^{nR_1} \rfloor\}$ and $\{1, \ldots, \lfloor 2^{nR_2} \rfloor\}$, where n denotes the block-length and R_1 and R_2 the rates of transmission.

The transmission takes place over a memoryless discrete-time Gaussian broadcast channel (BC). The time-t received symbols corresponding to the transmitted symbol x_t are thus

$$Y_{1,t} \quad = \quad x_t + Z_{1,t} \quad \text{and} \quad Y_{2,t} = x_t + Z_{2,t},$$

where $\{Z_{1,t}\}$ and $\{Z_{2,t}\}$ are independent sequences of independent and identically distributed zero-mean Gaussian random variables of variances $\sigma_1^2 > 0$ and $\sigma_2^2 > 0$.

The transmitter is assumed to have feedback from Receiver 2. Thus, it can compute X_t not only as a function of the messages M_1 and M_2 but also of the previous channel outputs $Y_2^{t-1} \triangleq (Y_{2,1}, \ldots, Y_{2,t-1})$:

$$X_t = x_t^{(n)} \left(M_1, M_2, Y_2^{t-1} \right).$$

The channel inputs are subject to an average power constraint $\frac{1}{n} \mathsf{E} \left[\sum_{t=1}^n X_t^2 \right] \leq P$. The two receivers decode their intended message based on the observed output sequences $Y_1^n = (Y_{1,1}, \ldots, Y_{1,n})$ and $Y_2^n = (Y_{2,1}, \ldots, Y_{2,n})$, respectively. In the described setup achievability of a pair of nonnegative rates (R_1, R_2) is defined as usual.

II. Results

Theorem 1. *A rate pair (R_1, R_2) is achievable whenever*

$$R_1 \leq \frac{1}{2\eta} \log \left(\frac{\det \left(\mathsf{K}_{U1} + \mathsf{K}_{U2} + BB^\mathsf{T} \sigma_2^2 + \mathsf{I}_\eta \sigma_1^2 \right)}{\det \left(\mathsf{K}_{U2} + BB^\mathsf{T} \sigma_2^2 + \mathsf{I}_\eta \sigma_1^2 \right)} \right)$$

$$R_2 \leq \frac{1}{2\eta} \log \left(\frac{\det \left(\mathsf{K}_{U2} + (B + \mathsf{I}_\eta)(B + \mathsf{I}_\eta)^\mathsf{T} \sigma_2^2 \right)}{\det(\mathsf{I}_\eta \sigma_2^2)} \right)$$

for some positive integer η, two positive semi-definite $\eta \times \eta$-matrices $\mathsf{K}_{U1}, \mathsf{K}_{U2}$, and a strictly lower-triangular $\eta \times \eta$-matrix B such that

$$\operatorname{tr} \left(\mathsf{K}_{U1} + \mathsf{K}_{U2} + \mathsf{B}\mathsf{B}^\mathsf{T} \sigma_2^2 \right) \leq \eta P,$$

and where I_η denotes the $\eta \times \eta$ identity matrix.

Theorem 2. *A rate-pair (R_1, R_2) is achievable whenever*

$$R_1 \leq \frac{1}{4\pi} \int_0^{2\pi} \log \left(1 + \frac{\mathcal{S}_{U1}(\omega)}{\mathcal{S}_{U2}(\omega) + |H(\omega)|^2 \sigma_2^2 + \sigma_1^2} \right) \mathrm{d}\omega$$

$$R_2 \leq \frac{1}{4\pi} \int_0^{2\pi} \log \left(\frac{\mathcal{S}_{U2}(\omega)}{\sigma_2^2} + |H(\omega) + 1|^2 \right) \mathrm{d}\omega$$

for some strictly causal filter with Fourier-Transform H and some power-spectral densities $\mathcal{S}_{U1}$ and $\mathcal{S}_{U2}$ that satisfy:

$$\frac{1}{2\pi} \int_0^{2\pi} \left(\mathcal{S}_{U1}(\omega) + \mathcal{S}_{U2}(\omega) + |H(\omega)|^2 \sigma_2^2 \right) \mathrm{d}\omega \leq P.$$

The region in Theorem 2 is a subset of the region in Theorem 1. The reason for also presenting the possibly smaller region in Theorem 2 is that for this second region it is possible to partly determine the optimal parameters (see the following Remark 3), which seems out of reach for the first region.

Remark 3. *For a specific choice of the filter H it is possible to derive the optimal power-spectral densities $\mathcal{S}_{U1}^*$ and $\mathcal{S}_{U2}^*$. For brevity the description of these waterfilling-type solutions is omitted.*

Specializing the achievable region in Theorem 2 for different values of α to the choice of parameters $H(\omega) = -\frac{\sqrt{\frac{\alpha^2 P^2}{\sigma^2(\alpha P + \sigma^2)}} e^{-\mathrm{i}\omega}}{1 - \sqrt{\frac{\sigma^2}{\alpha P + \sigma^2}} e^{-\mathrm{i}\omega}}$, $\mathcal{S}_{U2}(\omega) = 0$, and the optimal $\mathcal{S}_{U1}^*(\omega)$ recovers the achievable region in [1]. However, in general this choice of parameters is not optimal, and our achievable region in Theorem 2 is larger than the achievable region in [1]. In fact, even the achievable region in Theorem 1 with parameter $\eta = 2$ can be larger than the region in [1]. The same techniques can be applied for the double sided feedback. This direction is under investigation now.

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References

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