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The State-Dependent Semideterministic Broadcast Channel

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Abstract—**We derive the capacity region of the state-dependent semideterministic broadcast channel with noncausal state-information at the transmitter. In this broadcast channel one of the outputs is a deterministic function of the channel input and the channel state, and the state is assumed to be known noncausally to the transmitter but not to the receivers.**

I. INTRODUCTION

We characterize the capacity region of the discrete, memoryless, state-dependent, semideterministic broadcast channel. Such a channel has a single transmitting node, two receiving nodes, and an internal state, all of which are assumed to take value in finite sets. One of the receiving nodes observes a symbol Y that is a deterministic function of the transmitted symbol x and the state S

$$Y = f(x, S), \tag{1a}$$

and the other receiving node observes a symbol Z which is random: conditional on the input being x and the state being s, the probability that it is z is $W(z|x, s)$:

$$\Pr(Z = z|X = x, S = s) = W(z|x, s). \tag{1b}$$

The state sequence $\mathbf{S}$ is assumed to be independent and identically distributed (IID) according to some law $P_S(\cdot)$

$$\Pr(S = s) = P_S(s) \tag{1c}$$

and to be revealed to the encoder in a noncausal way: all future values of the state are revealed to the transmitter before transmission begins.

We consider a scenario where the encoder wishes to convey *two private messages*: $M_y \in \{1, \ldots, 2^{nR_y}\}$ to the deterministic receiver, and $M_z \in \{1, \ldots, 2^{nR_z}\}$ to the nondeterministic receiver, where R_y and R_z denote the *rates* (in bits per channel use) of data transmission to the deterministic and nondeterministic receivers, respectively. The messages M_y and M_z are assumed to be independent and uniformly distributed. As for the broadcast channel without a state [1], [2], we define the *capacity region* of this channel as the closure of all rate-pairs that are achievable in the sense that the probability that at least one of the receivers decodes its message incorrectly can be made arbitrarily close to zero.

The main result of this paper is a single-letter characterization of the capacity region:

Theorem 1. *The capacity region of the channel* (1) *when the states are known noncausally to the transmitter is the convex closure of the union of rate-pairs* (R_y, R_z) *satisfying*

$$R_y < H(Y|S) \tag{2a}$$

$$R_z < I(U; Z) - I(U; S) \tag{2b}$$

$$R_y + R_z < H(Y|S) + I(U; Z) - I(U; S, Y) \tag{2c}$$

over all joint distribution on (X, Y, Z, S, U) *whose marginal is the given state distribution* P_S *and under which, conditional on* X *and* S*, the channel outputs* Y *and* Z *are drawn according to the channel law* (1) *independently of* U*:*

$$\begin{aligned} &P_{XYZSU}(x, y, z, s, u) \\ &\quad = P_S(s)P_{XU|S}(x, u|s)\mathbf{1}\{y = f(x, s)\}W(z|x, s). \end{aligned} \tag{3}$$

Here $\mathbf{1}\{\cdot\}$ *denotes the indicator function. Moreover, the capacity region remains the same even if the state sequence is revealed to the deterministic receiver, i.e., if we replace* $f(\cdot, \cdot)$ *with the mapping* $(x, s) \mapsto \big(f(x, s), s\big)$*.*

We further have the following cardinality bound on the auxilliary random variable U:

Proposition 1. *To exhaust the capacity region of the channel* (1)*, we may restrict the auxilliary random variable* U *in* (2) *to take value in a set* $\mathcal{U}$ *satisfying*

$$|\mathcal{U}| \leq |\mathcal{X}| \cdot |\mathcal{S}| + 2, \tag{4}$$

where $\mathcal{X}$ *denotes the input alphabet, and where* $\mathcal{S}$ *denotes the state alphabet.*

The proof of Proposition 1 is omitted.

State-dependent broadcast channels were considered before [3]–[5]. Steinberg [3] studied the *degraded* state-dependent broadcast channel with causal and with noncausal side-information at the transmitter. He derived the capacity region for the causal case, but for the noncausal case his outer and inner bounds do not coincide. Steinberg and Shamai [4] then derived an inner bound for general (not necessarily degraded) state-dependent broadcast channels with noncausal

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The value of $\mathbf{1}\{\text{statement}\}$ is 1 if the statement is true and is 0 otherwise.

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side-information. This inner bound is based on Marton's inner bound for broadcast channels without states [6] and on Gel'fand-Pinsker coding [7]. In fact, the direct part of our Theorem 1 can be deduced from [4] with a proper choice of the auxiliary random variables (see Section II-A). Some special cases of Theorem 1 were solved by Khosravi-Farsani and Marvasti [5]: the *fully* deterministic case, the case where the states are known to the nondeterministic receiver, and the case where the channel is degraded so $(X,S)\multimap\!\!-Y\multimap\!\!-Z$ forms a Markov chain. However, capacity regions of most state-dependent broadcast channels are still unknown.

Much work has been done on broadcast channels *without states* [8]. Our work can be considered as an extension of previous works on deterministic broadcast channels (solved by Gel'fand, Marton and Pinsker [9]–[11]) and on semideterministic broadcast channels (solved by Gel'fand and Pinsker [12]). These results can be found in [2].

In the rest of this paper we prove the direct and converse parts of Theorem 1 in Sections II and III, respectively.

II. DIRECT PART

In this section we prove the direct part of Theorem 1. One way to do this is to use [4, Theorem 1] with a judicious choice of the auxiliary random variables, as we propose in Section II-A. For completeness and simplicity, we also sketch a self-contained proof in Section II-B. The complete version of the self-contained proof can be found in [13].

A. *Proof based on [4]*

It was shown in [4, Theorem 1] that the capacity region of any (not necessarily semideterministic) state-dependent broadcast channel with noncausal side-information at the transmitter contains the convex closure of the union of rate-pairs (R_y,R_z) satisfying

$$R_y \le I(U_0,U_y;Y) - I(U_0,U_y;S) \tag{5a}$$

$$R_z \le I(U_0,U_z;Z) - I(U_0,U_z;S) \tag{5b}$$

$$\begin{aligned}R_y + R_z \le &-\big[\max\{I(U_0;Y),I(U_0;Z)\} - I(U_0;S)\big]^+\\ &+ I(U_0,U_y;Y) - I(U_0,U_y;S) + I(U_0,U_z;Z)\\ &- I(U_0,U_z;S) - I(U_y;U_z|U_0,S),\end{aligned} \tag{5c}$$

where the union is over all joint distributions of (X,Y,Z,S,U_0,U_y,U_z) whose marginal is P_S; that satisfy the Markov condition

$$(U_0,U_y,U_z)\multimap\!\!-(X,S)\multimap\!\!-(Y,Z); \tag{6}$$

and under which the conditional law of (Y,Z) given (X,S) is that of the given channel.

For the semideterministic channel, we choose the auxiliary random variables in (5) as follows:

$$U_0 = 0 \quad \text{(deterministic)} \tag{7a}$$

$$U_y = Y \tag{7b}$$

$$U_z = U. \tag{7c}$$

Note that the Markov condition (6) is satisfied because Y is a deterministic function of (X,S) and because in Theorem 1 we restrict U to be such that $U\multimap\!\!-(X,S)\multimap\!\!-(Y,Z)$. With this choice of U_0, U_y, and U_z, (5) reduces to (2).

B. *Self-Contained Proof*

We next sketch a self-contained proof of the direct part of Theorem 1. (See [13] for the complete proof.) Like [4, Theorem 1], our proof is based on Marton's inner bound for general broadcast channels [6], [14] and on Gel'fand-Pinsker coding [7].

First note that the joint distribution (3) can also be written as

$$\begin{aligned}&P_{XYZSU}(x,y,z,s,u)\\ &\quad= P_S(s)P_{YU|S}(y,u|s)P_{X|YSU}(x|y,s,u)W(z|x,s)\end{aligned} \tag{8}$$

with the additional requirement that

$$y = f(x,s). \tag{9}$$

Further note that, when P_{YSU} is fixed, all the terms on the RHS of (2) are fixed, except for $I(U;Z)$, which is convex in $P_{X|YUS}$. Since $I(U;Z)$ only appears with a positive sign on the RHS of (2), it follows that the union over all joint distributions of the form (2) can be replaced by a union only over those where x is a deterministic function of (y,u,s), i.e., of the form

$$\begin{aligned}&P_{XYZSU}(x,y,z,s,u)\\ &\quad= P_S(s)P_{YU|S}(y,u|s)\mathbf{1}\{x=g(y,u,s)\}W(z|x,s)\end{aligned} \tag{10}$$

for some $g\colon (y,u,s)\mapsto x$ (and subject to (9)). We shall thus only establish the achievability of rate-pairs that satisfy (2) for some distribution of the form (10).

Choose a stochastic kernel $P_{YU|S}$ and a mapping $g\colon (y,u,s)\mapsto x$ which, combined with P_S and the channel law, determines the joint distribution (10) for which (9) is satisfied. For a given block-length n, we generate a random code as follows:

Codebook:
Generate 2^{nR_y} y-bins, each containing $2^{n\tilde{R}_y}$ y-tuples where the l_y-th y-tuple in the m_y-th bin

$$\mathbf{y}(m_y,l_y), \quad m_y \in \{1,\dots,2^{nR_y}\},\ l_y \in \{1,\dots,2^{n\tilde{R}_y}\}$$

is generated IID according to P_Y (the Y-marginal of (10)) independently of the other y-tuples. Additionally, generate 2^{nR_z} u-bins, each containing $2^{n\tilde{R}_z}$ u-tuples, where the l_z-th u-tuple in the m_z-th u-bin

$$\mathbf{u}(m_z,l_z), \quad m_z \in \{1,\dots,2^{nR_z}\},\ l_z \in \{1,\dots,2^{n\tilde{R}_z}\}$$

is drawn IID according to P_U (the U-marginal of (10)) independently of the other u-tuples and of the y-tuples.

Encoder:
To send message $m_y \in \{1,\dots,2^{R_y}\}$ to the deterministic receiver and message $m_z \in \{1,\dots,2^{R_z}\}$ to the nondeterministic receiver, look for a y-tuple $\mathbf{y}(m_y,l_y)$ in y-bin m_y and a u-tuple $\mathbf{u}(m_z,l_z)$ in u-bin m_z such that

$\big(\mathbf{y}(m_y,l_y),\mathbf{u}(m_z,l_z)\big)$ is jointly typical with the state sequence $\mathbf{s}$. If such a pair can be found, send

$$\mathbf{x} = g\big(\mathbf{y}(m_y,l_y),\mathbf{u}(m_z,l_z),\mathbf{s}\big), \tag{11}$$

where in the above $g(\mathbf{y},\mathbf{u},\mathbf{s})$ denotes the application of the function $g(y,u,s)$ componentwise. In this case the sequence received by the deterministic receiver will be $\mathbf{y}(m_y,l_y)$. Otherwise send an arbitrary codeword.

Deterministic decoder:

Try to find the *unique* y-bin, say m'_y, that contains the received sequence $\mathbf{y}$ and output its number m'_y. If there is more than one such bin, declare an error.

Nondeterministic decoder:

Try to find the *unique* u-tuple $\mathbf{u}(m'_z,l'_z)$ that is jointly typical with the received sequence $\mathbf{z}$ and output its bin number m'_z. If more than one or no such $\mathbf{u}$ can be found, declare an error.

We next analyze the error probability of the above coding scheme. There are three types of errors:

Deterministic decoder errs.

This happens only if there is more than one bin that contains the received $\mathbf{y}$. This probability tends to zero as n tends to infinity provided

$$R_y + \tilde{R}_y < H(Y). \tag{12}$$

Nondeterministic decoder errs.

This happens if either the u-tuple $\mathbf{u}(m_z,l_z)$ is not jointly typical with the received z-tuple, or if a different u-tuple happens to be jointly typical with the received z-tuple. The probability of the former case tends to zero as n tends to infinity by the Markov Lemma [2]. The probability of the latter case tends to zero as n tends to infinity provided

$$R_z + \tilde{R}_z < I(U;Z). \tag{13}$$

Encoder errs.

This happens only if there is no $(l_y,l_z) \in \{1,\ldots,2^{n\tilde{R}_y}\} \times \{1,\ldots,2^{n\tilde{R}_z}\}$ such that $\big(\mathbf{y}(m_y,l_y),\mathbf{u}(m_z,l_z)\big)$ is jointly typical with the state sequence $\mathbf{s}$. Using the Multivariate Covering Lemma [2, Lemma 8.2], we obtain that this error probability tends to zero as n tends to infinity provided

$$\tilde{R}_y > I(Y;S) \tag{14a}$$
$$\tilde{R}_z > I(U;S) \tag{14b}$$
$$\tilde{R}_y + \tilde{R}_z > H(Y) + H(U) + H(S) - H(Y,U,S). \tag{14c}$$

Summarizing (12), (13) and (14) we conclude that the above coding scheme has vanishing error probability as n tends to infinity for all (R_y,R_z) satisfying (2). By time-sharing we further achieve the convex hull of all rate-pairs satisfying (2) for joint distributions of the form (10). This concludes the proof of the direct part of Theorem 1.

III. CONVERSE PART

Our proof of the converse part of Theorem 1 employs ideas from Nair and El Gamal's outer bound [15] and of Gel'fand and Pinkser's converse for the state-dependent single-user channel [7], but it also contains some new elements.

We shall show that, even if the state sequence $\mathbf{S}$ is revealed to the deterministic receiver (which observes $\mathbf{Y}$), any achievable rate-pair must be in the convex closure of the union of rate-pairs satisfying (2). Given any code of block-length n, we first derive a bound on R_y:

$$\begin{aligned}
nR_y &= H(M_y) && (15)\\
&\le I(M_y;Y^n,S^n) + n\epsilon_n && (16)\\
&= I(M_y;Y^n|S^n) + n\epsilon_n && (17)\\
&= \sum_{i=1}^{n} I(M_y;Y_i|Y^{i-1},S^n) + n\epsilon_n && (18)\\
&\le \sum_{i=1}^{n} H(Y_i|Y^{i-1},S^n) + n\epsilon_n && (19)\\
&\le \sum_{i=1}^{n} H(Y_i|S_i) + n\epsilon_n, && (20)
\end{aligned}$$

where ϵ_n is a function of n which decays to zero as n goes to infinity. Here, (16) follows from Fano's Inequality; (17) because M_y and S^n are independent; (18) from the chain rule; (19) by dropping negative terms; and (20) because conditioning cannot increase entropy.

We next bound R_z as in [7]:

$$\begin{aligned}
nR_z &= H(M_z) && (21)\\
&\le I(M_z;Z^n) + n\epsilon_n && (22)\\
&= \sum_{i=1}^{n} I(M_z;Z_i|Z^{i-1}) + n\epsilon_n && (23)\\
&= \sum_{i=1}^{n} I\left(M_z,S_{i+1}^n;Z_i\middle|\,Z^{i-1}\right)\\
&\quad - \sum_{i=1}^{n} I\left(S_{i+1}^n;Z_i\middle|\,M_z,Z^{i-1}\right) + n\epsilon_n && (24)\\
&= \sum_{i=1}^{n} I\left(M_z,S_{i+1}^n;Z_i\middle|\,Z^{i-1}\right)\\
&\quad - \sum_{i=1}^{n} I\left(Z^{i-1};S_i\,\middle|M_z,S_{i+1}^n\right) + n\epsilon_n && (25)\\
&= \sum_{i=1}^{n} I\left(M_z,S_{i+1}^n;Z_i\middle|\,Z^{i-1}\right)\\
&\quad - \sum_{i=1}^{n} I\left(M_z,Z^{i-1},S_{i+1}^n;S_i\right) + n\epsilon_n && (26)\\
&\le \sum_{i=1}^{n} I\left(M_z,Z^{i-1},S_{i+1}^n;Z_i\right)\\
&\quad - \sum_{i=1}^{n} I\left(M_z,Z^{i-1},S_{i+1}^n;S_i\right) + n\epsilon_n && (27)
\end{aligned}$$

$$= \sum_{i=1}^{n} I(V_i; Z_i) - I(V_i; S_i) + n\epsilon_n. \tag{28}$$

Here, (22) follows from Fano's Inequality; (23) and (24) from the chain rule; (25) from Csiszár's Identity [16]

$$\sum_{i=1}^{n} I\left(C_{i+1}^{n}; D_i \middle| D^{i-1}\right) = \sum_{i=1}^{n} I\left(D^{i-1}; C_i \middle| C_{i+1}^{n}\right); \tag{29}$$

(26) because S_i and (M_z, S^{i-1}) are independent; (27) from the chain rule and by dropping negative terms; and (28) by defining the auxiliary random variables

$$V_i \triangleq (M_z, Z^{i-1}, S_{i+1}^{n}), \quad i \in \{1, \dots, n\}. \tag{30}$$

We next bound the sum rate $R_y + R_z$:

$$\begin{aligned} n(R_y + R_z) &= H(M_y, M_z) &(31)\\ &= H(M_z) + H(M_y|M_z) &(32)\\ &\leq I(M_z; Z^n) + I(M_y; Y^n, S^n|M_z) + n\epsilon_n, &(33)\end{aligned}$$

where the last step follows from Fano's Inequality. Of the two mutual informations on the RHS of (33) we first bound $I(M_z; Z^n)$:

$$\begin{aligned} I(M_z; Z^n) &= \sum_{i=1}^{n} I(M_z; Z_i|Z^{i-1}) &(34)\\ &\leq \sum_{i=1}^{n} I(M_z, Z^{i-1}; Z_i) &(35)\\ &= \sum_{i=1}^{n} I\left(M_z, Z^{i-1}, S_{i+1}^{n}, Y_{i+1}^{n}; Z_i\right)\\ &\quad - \sum_{i=1}^{n} I\left(S_{i+1}^{n}, Y_{i+1}^{n}; Z_i \middle| M_z, Z^{i-1}\right) &(36)\\ &= \sum_{i=1}^{n} I\left(M_z, Z^{i-1}, S_{i+1}^{n}, Y_{i+1}^{n}; Z_i\right)\\ &\quad - \sum_{i=1}^{n} I\left(Z^{i-1}; S_i, Y_i \middle| M_z, S_{i+1}^{n}, Y_{i+1}^{n}\right) &(37)\\ &= \sum_{i=1}^{n} I\left(M_z, Z^{i-1}, S_{i+1}^{n}, Y_{i+1}^{n}; Z_i\right)\\ &\quad - \sum_{i=1}^{n} I\left(M_z, Z^{i-1}, S_{i+1}^{n}, Y_{i+1}^{n}; S_i, Y_i\right)\\ &\quad + \sum_{i=1}^{n} I\left(M_z, S_{i+1}^{n}, Y_{i+1}^{n}; S_i, Y_i\right). &(38)\end{aligned}$$

Here, (34), (35) and (36) follow from the chain rule; (37) by applying Csiszár's Identity (29) between (S^n, Y^n) and Z^n; and (38) again from the chain rule.

We next study the sum of the last term on the RHS of (38) and the second mutual information on the RHS of (33):

$$\begin{aligned} &\sum_{i=1}^{n} I\left(M_z, S_{i+1}^{n}, Y_{i+1}^{n}; S_i, Y_i\right) + I(M_y; Y^n, S^n|M_z)\\ &\quad = \sum_{i=1}^{n} I\left(M_z, S_{i+1}^{n}, Y_{i+1}^{n}; S_i, Y_i\right)\\ &\qquad + \sum_{i=1}^{n} I\left(M_y; S_i, Y_i \middle| M_z, S_{i+1}^{n}, Y_{i+1}^{n}\right) &(39)\\ &\quad = \sum_{i=1}^{n} I\left(M_y, M_z, S_{i+1}^{n}, Y_{i+1}^{n}; S_i, Y_i\right) &(40)\\ &\quad = \sum_{i=1}^{n} I\left(M_y, M_z, S_{i+1}^{n}, Y_{i+1}^{n}; S_i, Y_i\right)\\ &\qquad + \sum_{i=1}^{n} I\left(S^{i-1}; S_i, Y_i \middle| M_y, M_z, S_{i+1}^{n}, Y_{i+1}^{n}\right)\\ &\qquad - \sum_{i=1}^{n} I\left(S_{i+1}^{n}, Y_{i+1}^{n}; S_i \middle| M_y, M_z, S^{i-1}\right) &(41)\\ &\quad = \sum_{i=1}^{n} I\left(M_y, M_z, S^{i-1}, S_{i+1}^{n}, Y_{i+1}^{n}; S_i, Y_i\right)\\ &\qquad - \sum_{i=1}^{n} I\left(S_{i+1}^{n}, Y_{i+1}^{n}; S_i \middle| M_y, M_z, S^{i-1}\right) &(42)\\ &\quad = \sum_{i=1}^{n} I\left(M_y, M_z, S^{i-1}, S_{i+1}^{n}, Y_{i+1}^{n}; S_i, Y_i\right)\\ &\qquad - \sum_{i=1}^{n} I\left(M_y, M_z, S^{i-1}, S_{i+1}^{n}, Y_{i+1}^{n}; S_i\right) &(43)\\ &\quad = \sum_{i=1}^{n} I\left(M_y, M_z, S^{i-1}, S_{i+1}^{n}, Y_{i+1}^{n}; Y_i \middle| S_i\right) &(44)\\ &\quad = \sum_{i=1}^{n} H(Y_i|S_i). &(45)\end{aligned}$$

Here, (39) and (40) follow from the chain rule; (41) by applying Csiszár's Identity between (S^n, Y^n) and S^n; (42) from the chain rule; (43) because S_i and (M_y, M_z, S^{i-1}) are independent; (44) again from the chain rule; and (45) because, given (M_y, M_z, S^n), the channel inputs X^n are determined by the encoder, and hence Y^n are also determined, so

$$H\left(Y_i \middle| M_y, M_z, S^n, Y_{i+1}^{n}\right) = 0. \tag{46}$$

Combining (33), (38) and (45), using the definitions (30), and further defining

$$T_i \triangleq Y_{i+1}^{n}, \quad i \in \{1, \dots, n\}, \tag{47}$$

we obtain

$$\begin{aligned} n(R_y + R_z) &\leq \sum_{i=1}^{n} I(V_i, T_i; Z_i) - \sum_{i=1}^{n} I(V_i, T_i; S_i, Y_i)\\ &\quad + \sum_{i=1}^{n} H(Y_i|S_i) + n\epsilon_n. &(48)\end{aligned}$$

Summarizing (20), (28) and (48) and letting n go to infinity we obtain that any achievable rate-pair (R_y, R_z) must be contained in the convex closure of the union of rate-pairs satisfying

$$R_y < H(Y|S) \tag{49a}$$

$$R_z < I(V;Z) - I(V;S) \tag{49b}$$

$$R_y + R_z < H(Y|S) + I(V,T;Z) - I(V,T;S,Y) \tag{49c}$$

where, given (X,S), the outputs (Y,Z) are drawn according to the channel law (1) independently of the auxiliary random variables (V,T). To prove the converse part of Theorem 1, it remains to replace V and T with a single auxiliary random variable. I.e., it remains to find an auxiliary random variable U such that

$$I(V;Z) - I(V;S) \le I(U;Z) - I(U;S), \tag{50a}$$

$$H(Y|S) + I(V,T;Z) - I(V,T;S,Y) \le H(Y|S) + I(U;Z) - I(U;S,Y). \tag{50b}$$

In fact, as we shall see, either choosing U to be V will satisfy (50) or else choosing it to be (V,T) will satisfy (50). If we choose $U = V$, then (50a) is satisfied with equality, and the requirement (50b) becomes

$$I(T;Z|V) - I(T;S,Y|V) \le 0. \tag{51}$$

On the other hand, if we choose $U = (V,T)$, then (50b) is satisfied with equality, and the requirement (50a) becomes

$$I(T;Z|V) - I(T;S|V) \ge 0. \tag{52}$$

It remains to show that at least one of the requirements (51) and (52) must be satisfied: if it is (51), then we shall choose U as V, and it if is (52), then we shall choose U as (V,T). To this end we note that for all random variables T, Z, V, S, Y

$$I(T;Z|V) - I(T;S,Y|V) \le I(T;Z|V) - I(T;S|V), \tag{53}$$

because the RHS minus the left-hand side equals $I(T;Y|S,V)$ which is nonnegative. Therefore, *at least one of* (51) and (52) must be satisfied. We have thus shown that there must exist a U which satisfies both inequalities in (50), hence the bounds (49) can be relaxed to (2). This concludes the proof of the converse part of Theorem 1.

Remark. *This outer bound can be easily generalized to a broadcast channel that is not necessarily semideterministic. Such a channel is described by the transition law*

$$\Pr(Y = y, Z = z | X = x, S = s) = W(y, z|x, s). \tag{54}$$

The general outer bound states the following: if the state sequence $\mathbf{S}$ *is revealed noncausally to the transmitter, and is revealed to the receiver who observes* Y *but not to the receiver which observes* Z*, then the capacity region of this channel is contained in the convex closure of rate-pairs satisfying*

$$R_y < I(X;Y|S) \tag{55a}$$

$$R_z < I(U;Z) - I(U;S) \tag{55b}$$

$$R_y + R_z < I(X;Y|S) + I(U;Z) - I(U;S,Y) \tag{55c}$$

for joint distributions of the form

$$P_{XYZSU}(x,y,z,s,u) = P_S(s) P_{XU|S}(x,u|s) W(y,z|x,s). \tag{56}$$

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